Tetrahedral Meshing in the Wild

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OUTLINE

- Related Work
- Method
- Results
- Limitations

Related Work

- Background Grids
- Delaunay
  - Constrained Delaunay tetrahedralization
  - Restricted Delaunay tetrahedralization
- Variational meshing
- Surface envelope

Low element quality, "sliver" tetrahedra, heavy or over refinement,

Lacking of robustness, requiring initial starting points …

Method

Tetrahedralize arbitrary meshes without assumptions on mesh manifoldness, watertightness, absence of self-intersections etc.

Method

Input: triangle soup

a user-specified tolerance ε

a desired target edge length l

Output: an approximately constrained tetrahedral mesh

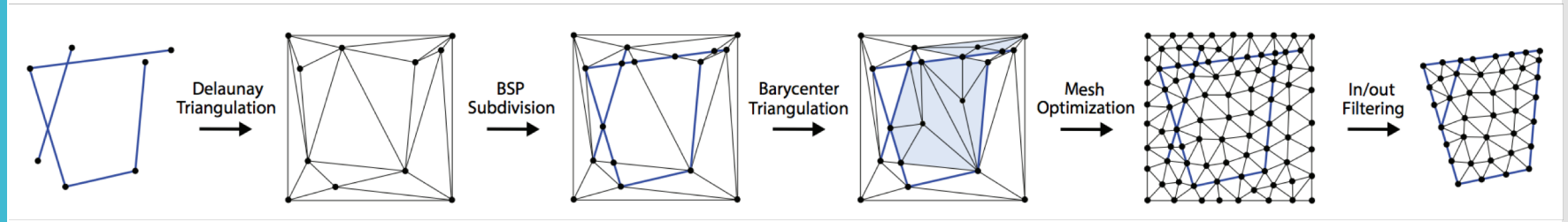
- Contains an approximation of the input set of triangles with in user-defined tolerance
- Has no inverted elements
- Edge length below user-defined bound l

Method

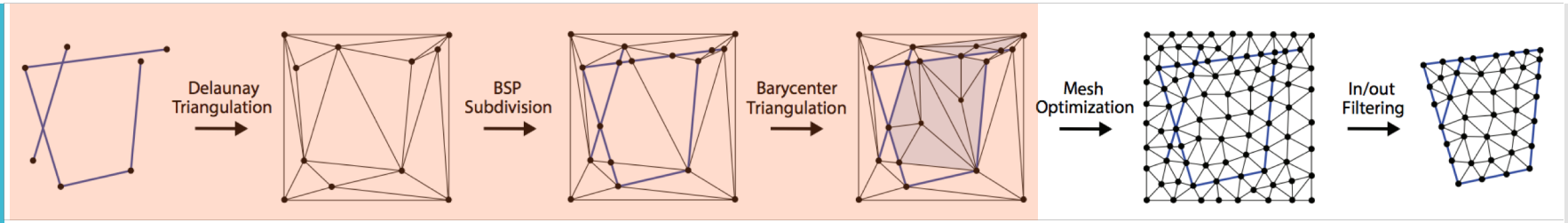
Phase 1: Generation of a valid mesh

Phase 2: Mesh Improvement

Phase 3: Interior volume extraction



Method

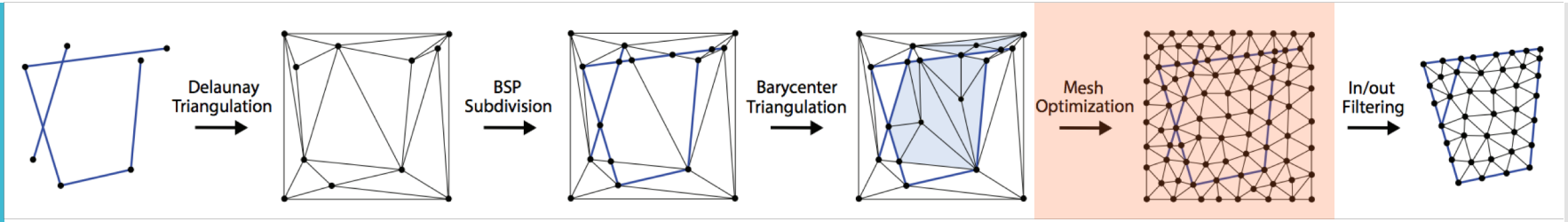


Phase 1: Generation of a valid mesh

- Create an initial, non-conforming tetrahedral mesh using Delaunay tetrahedralization
- BSP-Tree Subdivison
- Polyhedral mesh is converted to a tetrahedral mesh by adding a vertex at the barycenter, and connecting it to all triangular faces on the boundary

Self-intersection are naturally handled

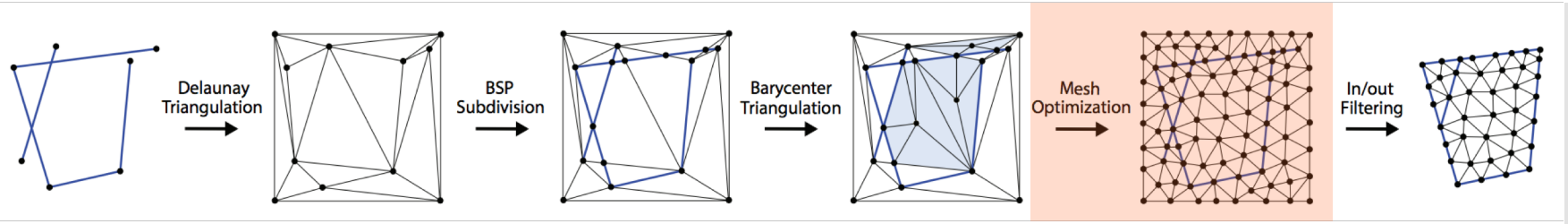
Method



Phase 2: Mesh Improvement

- Invariant
  - Disallow every operation introducing inverted tetrahedra
  - Only accept operations that keep the faces on the surface at a distance smaller than user-defined ε

Method



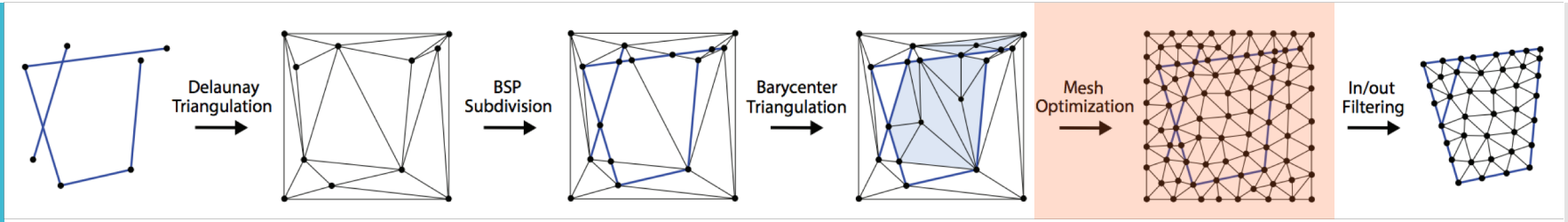
Phase 2: Mesh Improvement

- Quality
  - Minimize 3D conformal energy[1] which is well-correlated with many common measures of quality

$$\varepsilon = \sum_{t \in T} \frac{tr(J_t^T J_t)}{\det(J_t)^{\frac{2}{3}}}$$

1. Rabinovich et al. 2017

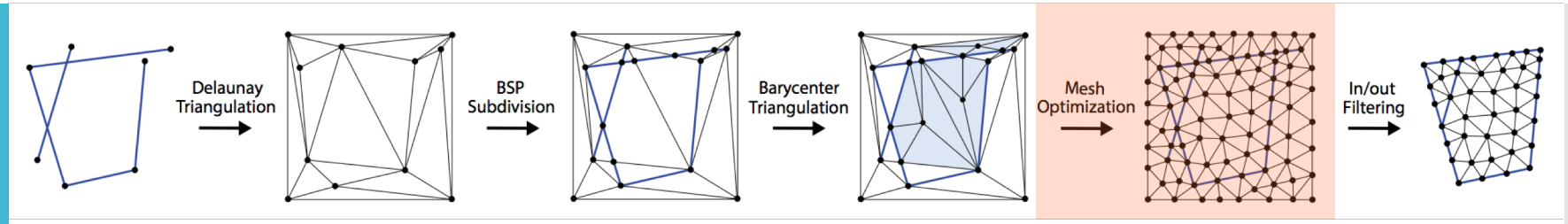
Method



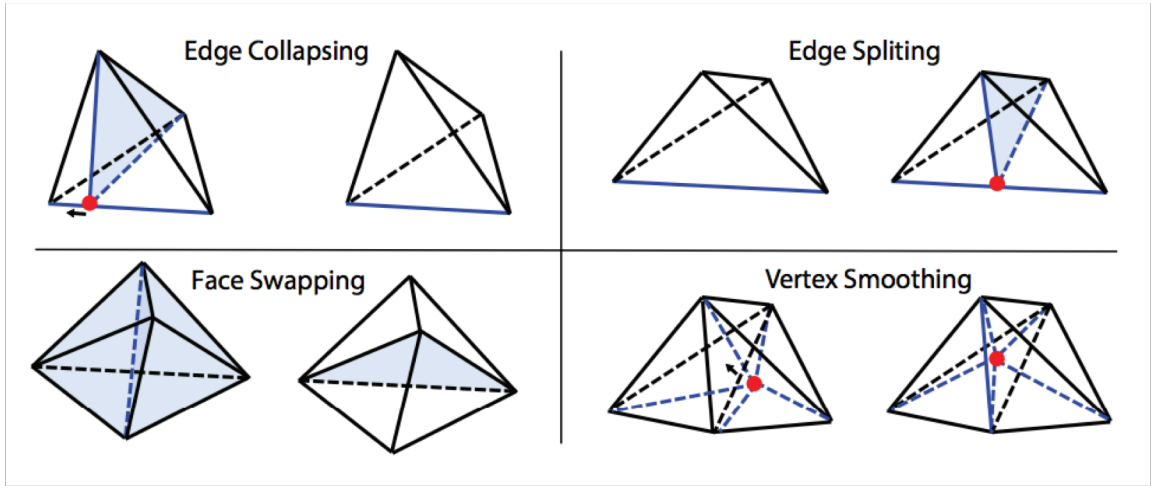
Phase 2: Mesh Improvement

- Local Operations
  - Edge splitting (refining)
  - Edge collapsing (coarsening)
  - Face swapping
  - Vertex smoothing

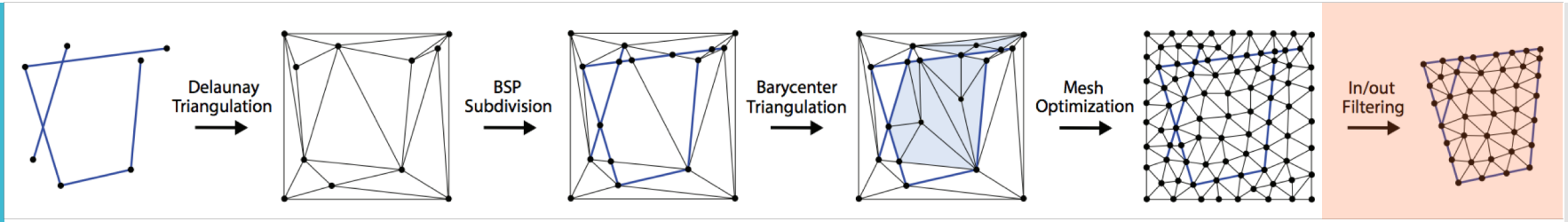
Method



Phase 2: Mesh Improvement



Method



Phase 3: Interior volume extraction

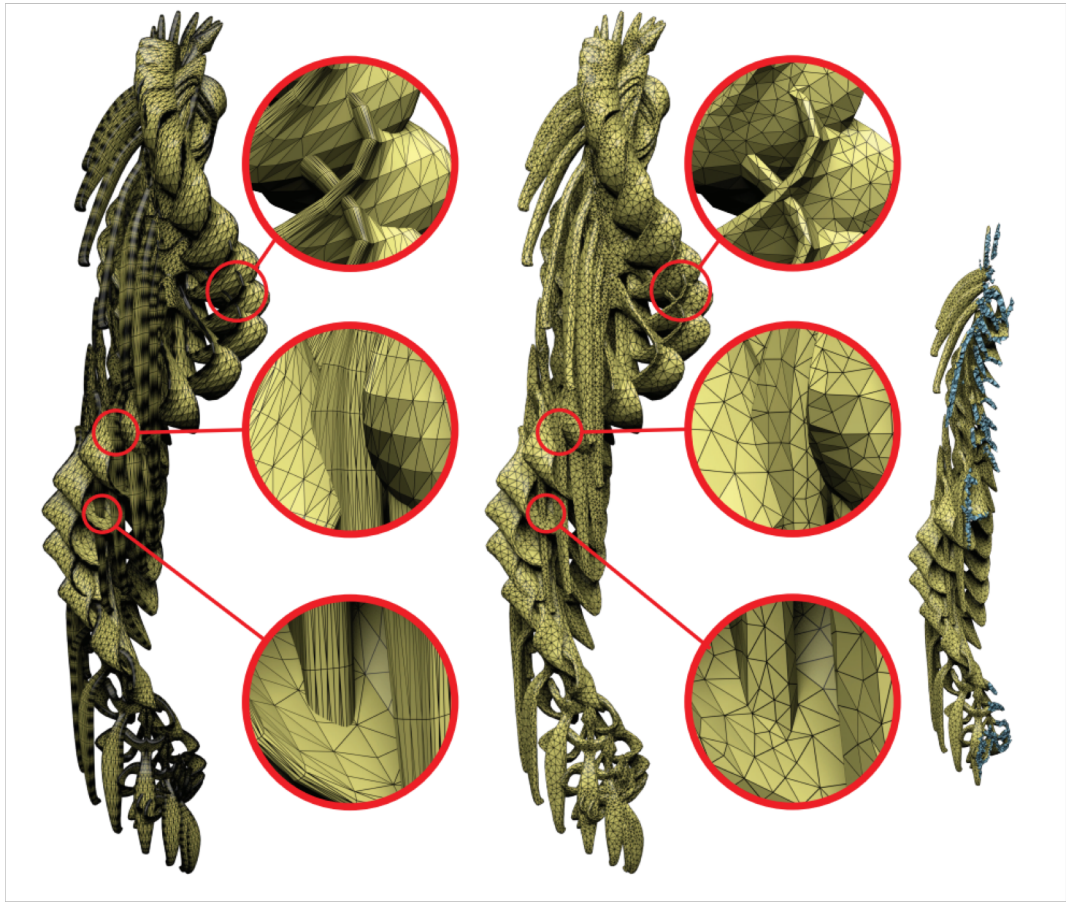
- Define an inside-outside function that can be used to extract an interior volume associated with the mesh[1]
  - Calculate the winding number of the centroid of each tetrahedron with respect to the embedded surface
    - If the winding number is smaller than 0.5, drop it

Both small gaps and large surface holes are filled according to the induced winding number field

1. Jacobson et al. 2013

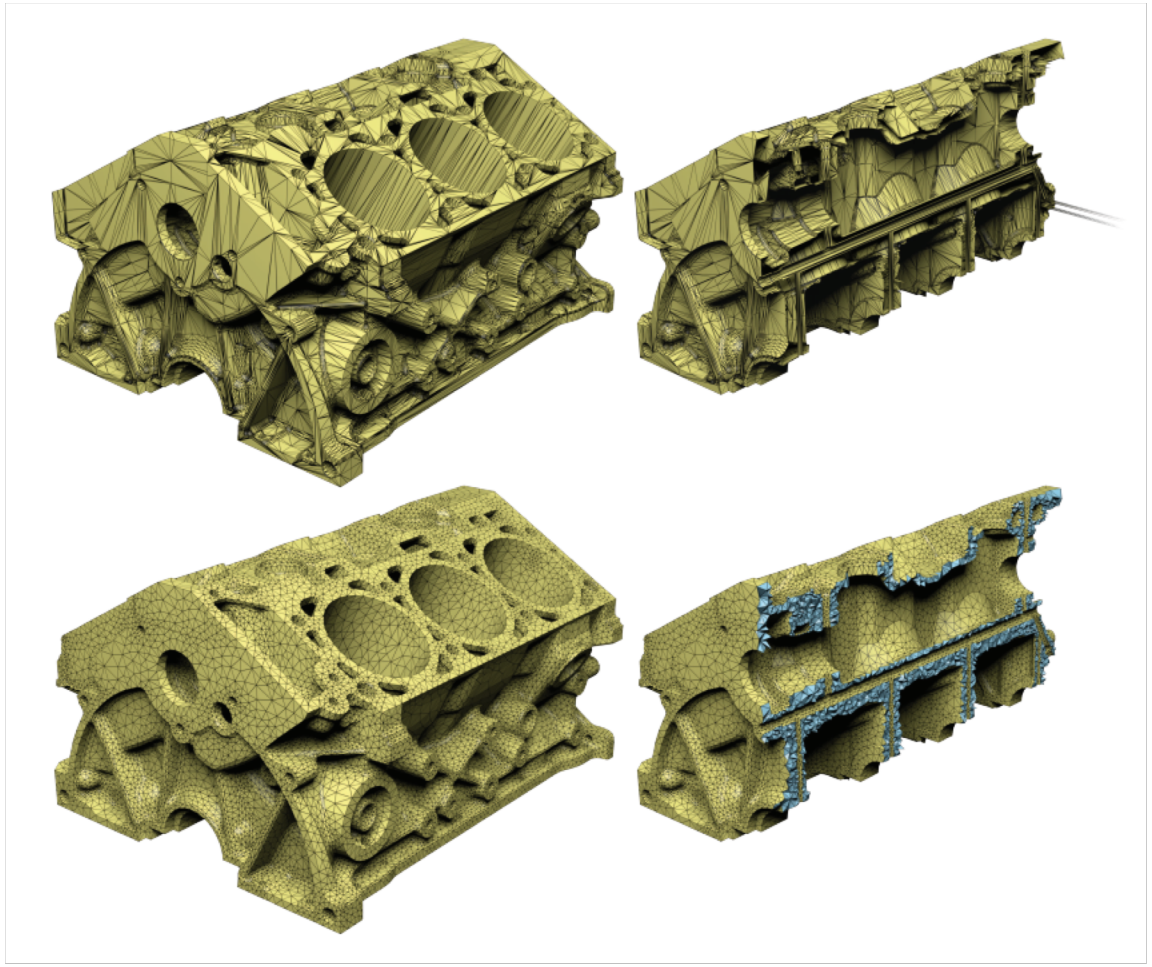
Results

Self-intersection



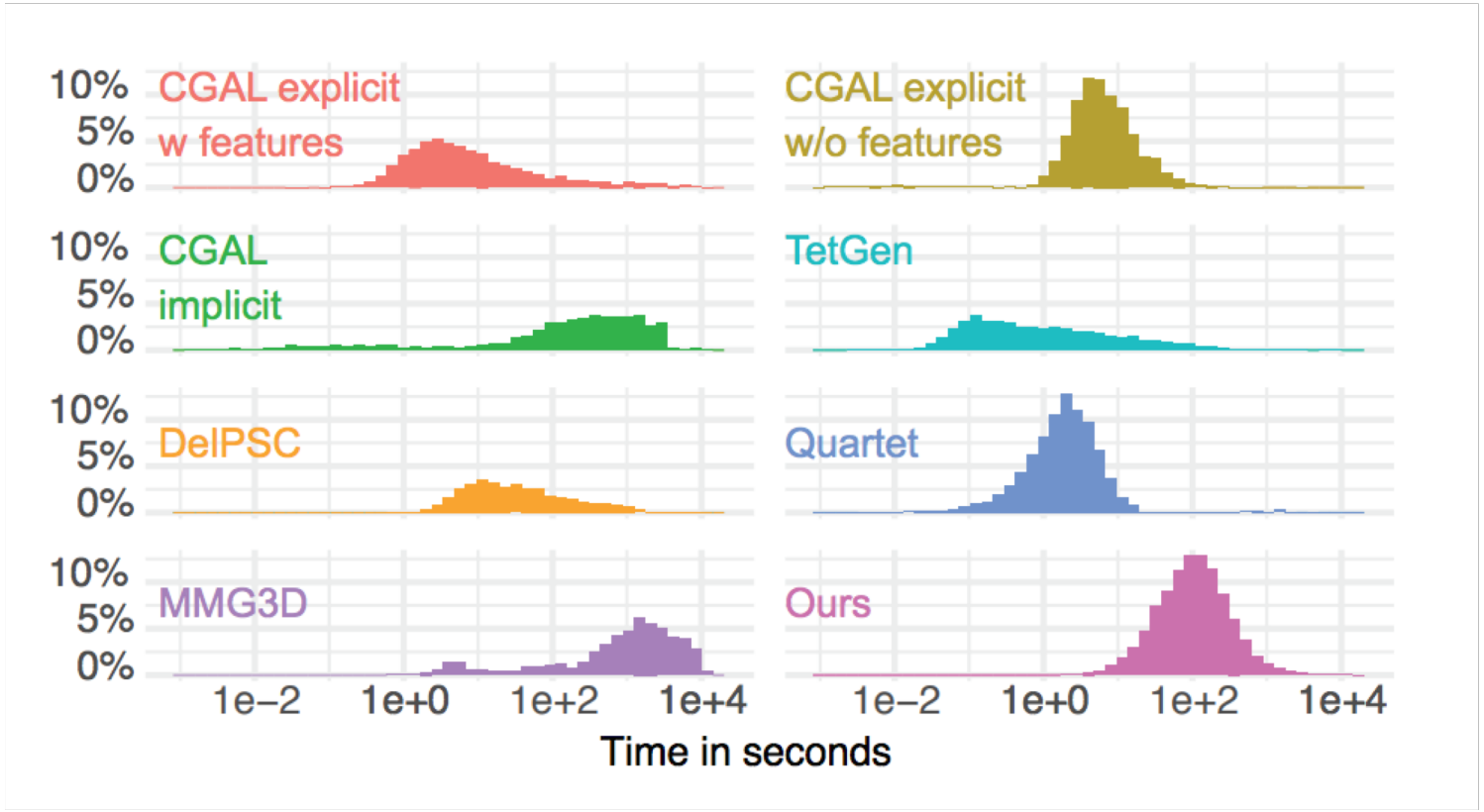
Results

High Quality



Results

Comparison of running time



Results

Comparison of mesh quality

Limitations

- Vertices could be displaces
  - Causing a straight line to zigzag
- Preservation of sharp features
- Limited to closed surfaces
- Slow

Thank you!