READING:

Presenter: Chenxi Liu

Fast Winding Numbers for Soups and Clouds

GAVIN BARILL, University of Toronto, Canada
NEIL G. DICKSON, Side Effects Software Inc., Canada
RYAN SCHMIDT, Gradientspace, Canada
DAVID I.W. LEVIN* and ALEC JACOBSON*, University of Toronto, Canada

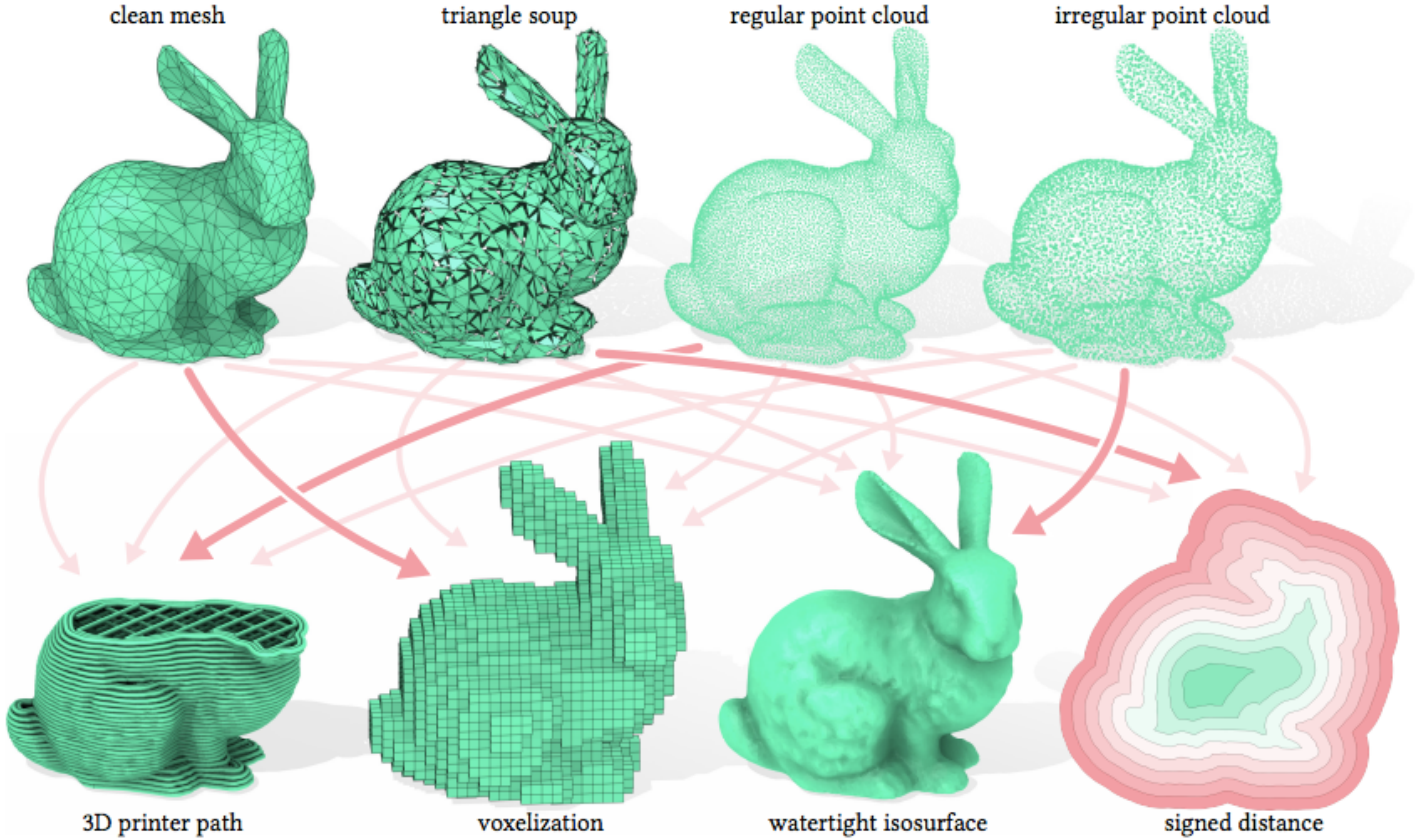

Fig. 1. In this paper, we further generalize the winding number to point clouds and propose a hierarchical algorithm for fast evaluation (up to 1000× speedup). This enables efficient answers to inside-outside queries for a wider class of shape representations (top) during a variety of tasks (bottom).

Winding Numbers?

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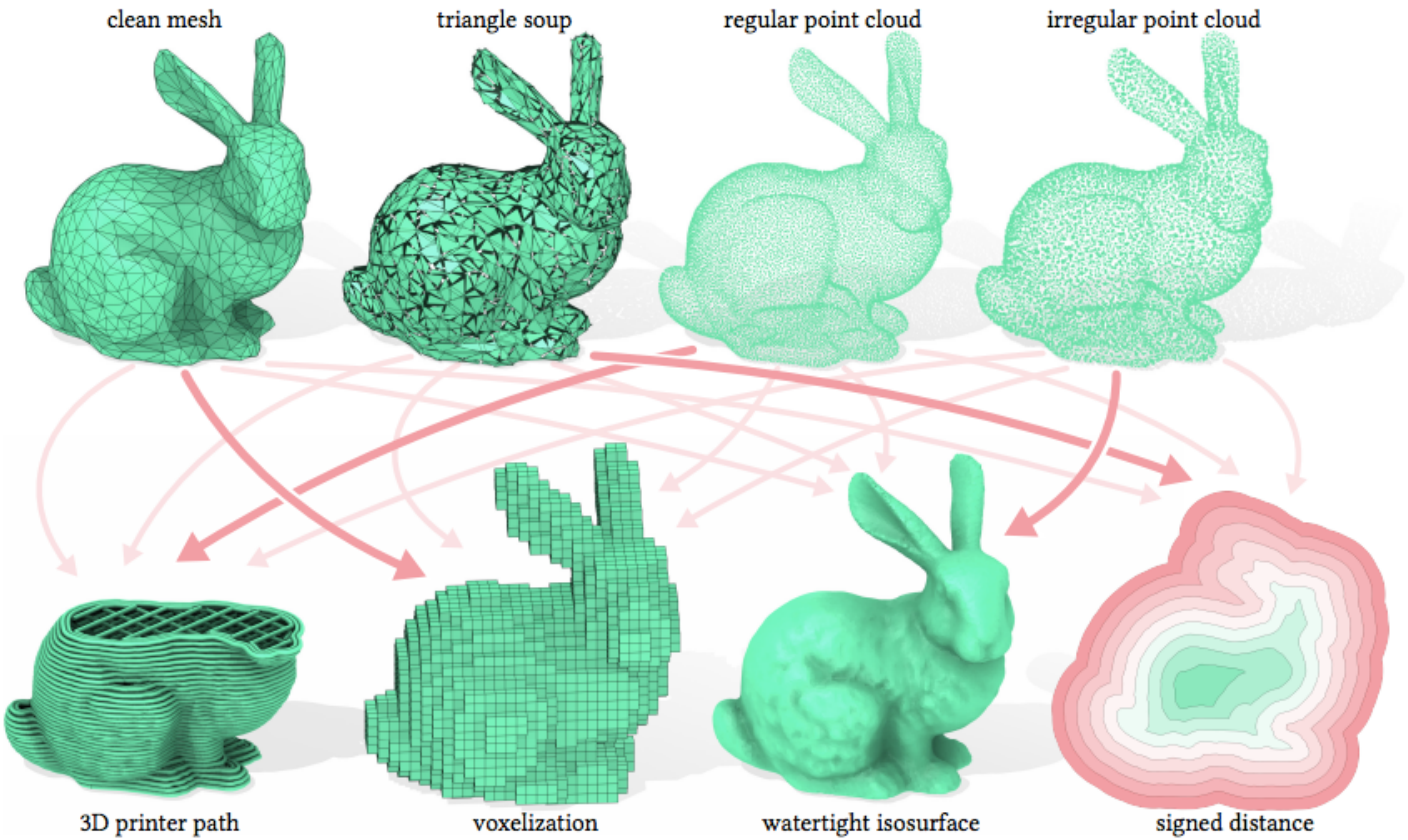

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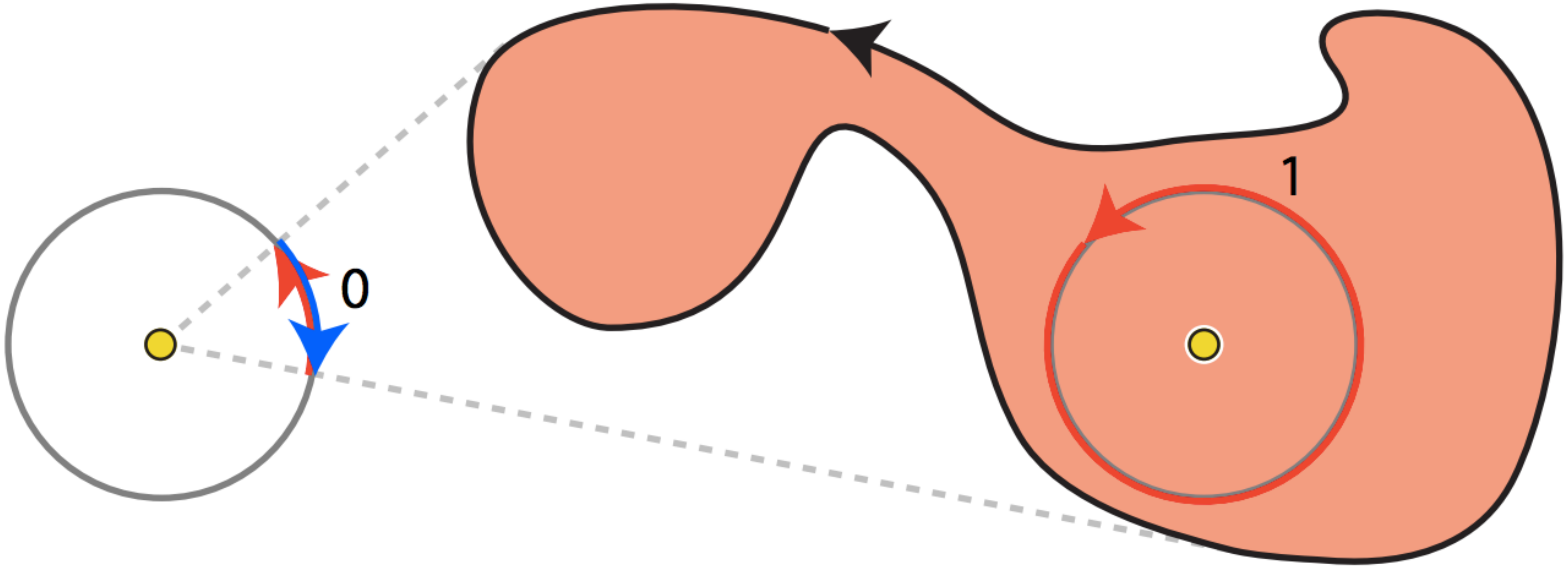
What are Winding Numbers?



At a given **point**, winding number is an **integer** defined as the **signed** length of the projection of a **closed** curve onto a circle divided by 2π.

Generalized Winding Numbers

The winding numbers is generalized to any **real** value defined for **open curves and surfaces** in this SIGGRAPH’13 paper:

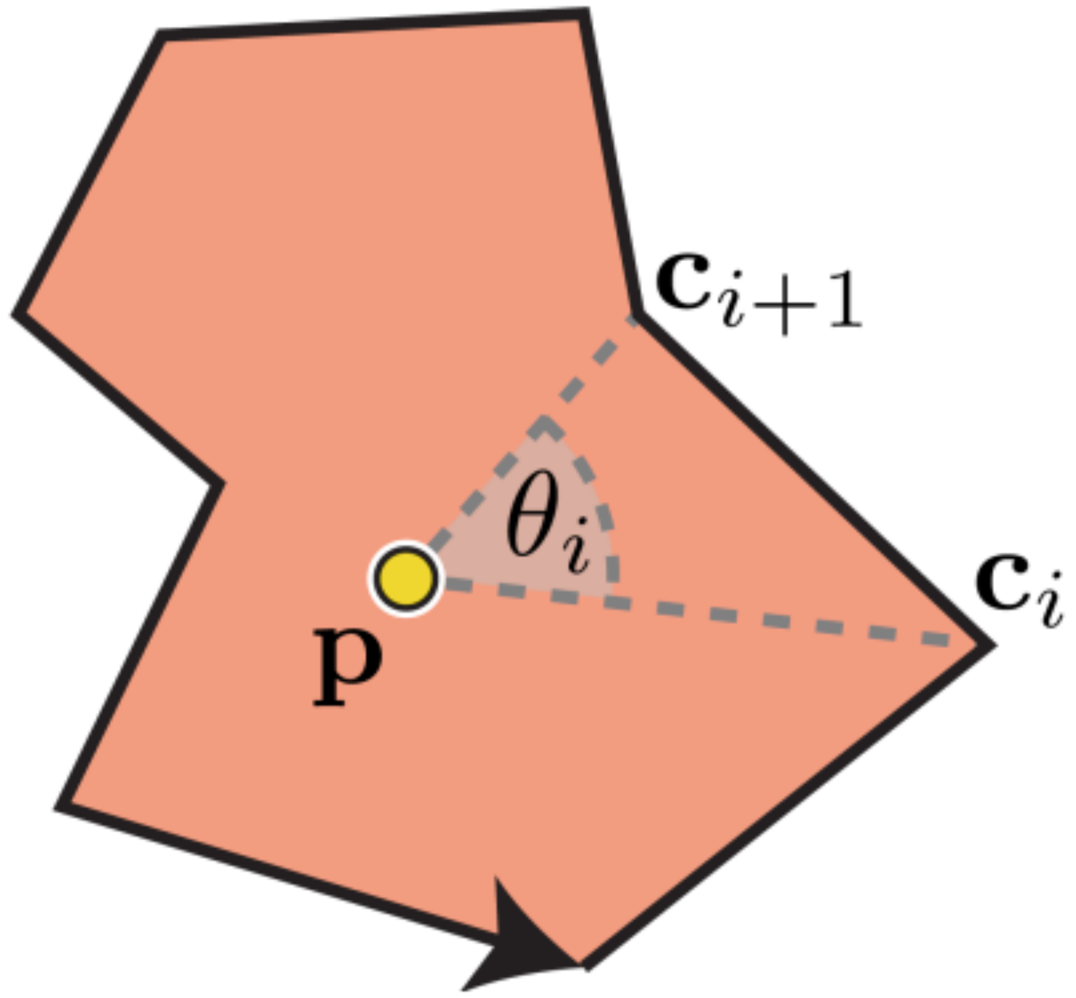
Robust Inside-Outside Segmentation using Generalized Winding Numbers

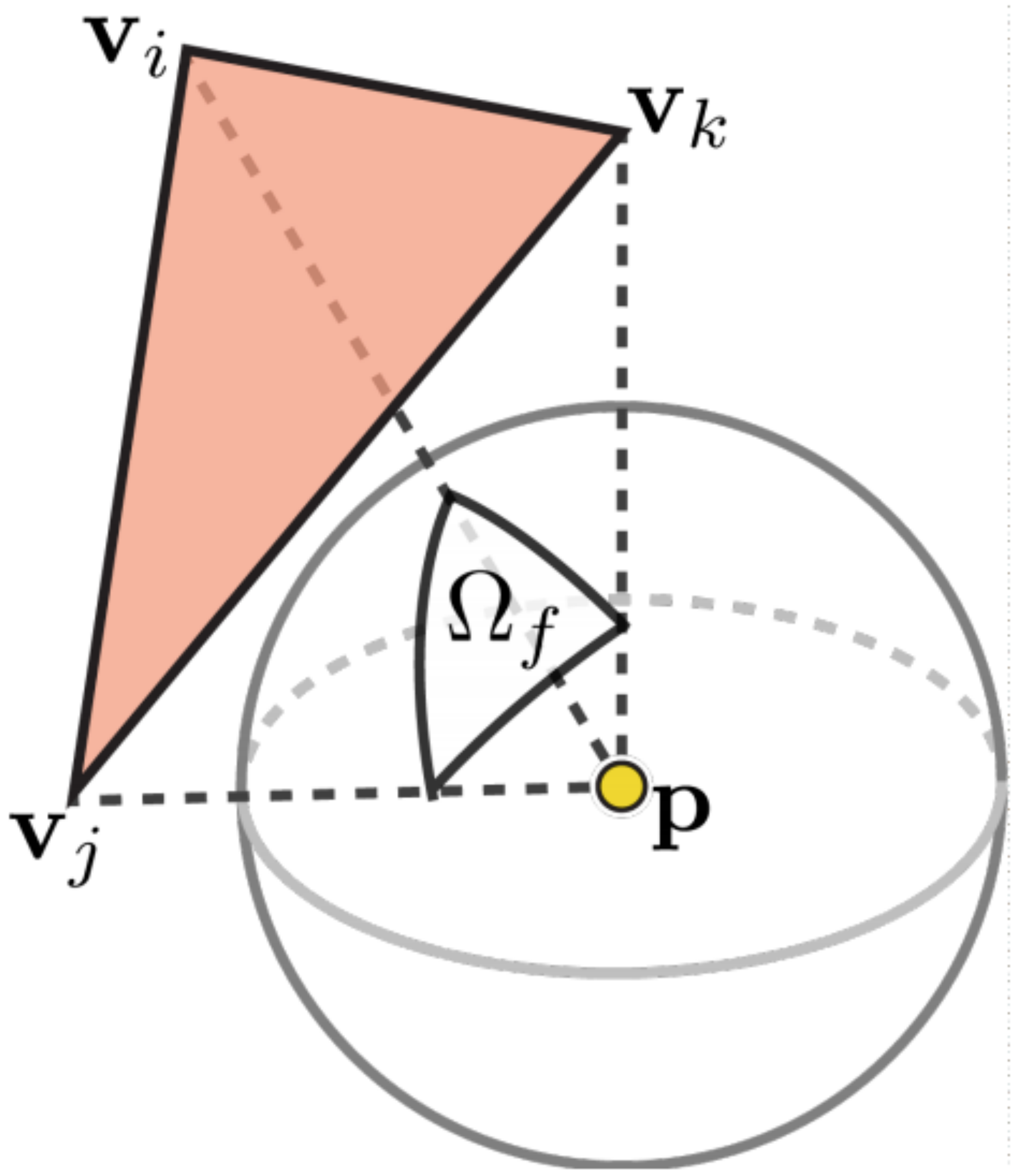
Alec Jacobson[1] Ladislav Kavan[2,1] Olga Sorkine-Hornung[1]

[1]ETH Zurich [2]University of Pennsylvania

Figure 1: *The* Big SigCat *input mesh has 3442 pairs of intersecting triangles (bright red), 1020 edges on open boundaries (dark red), 344 non-manifold edges (purple) and 67 connected components (randomly colored). On top of those problems, a SIGGRAPH logo shaped hole is carved from her side.*

Generalized Winding Numbers





Curve:

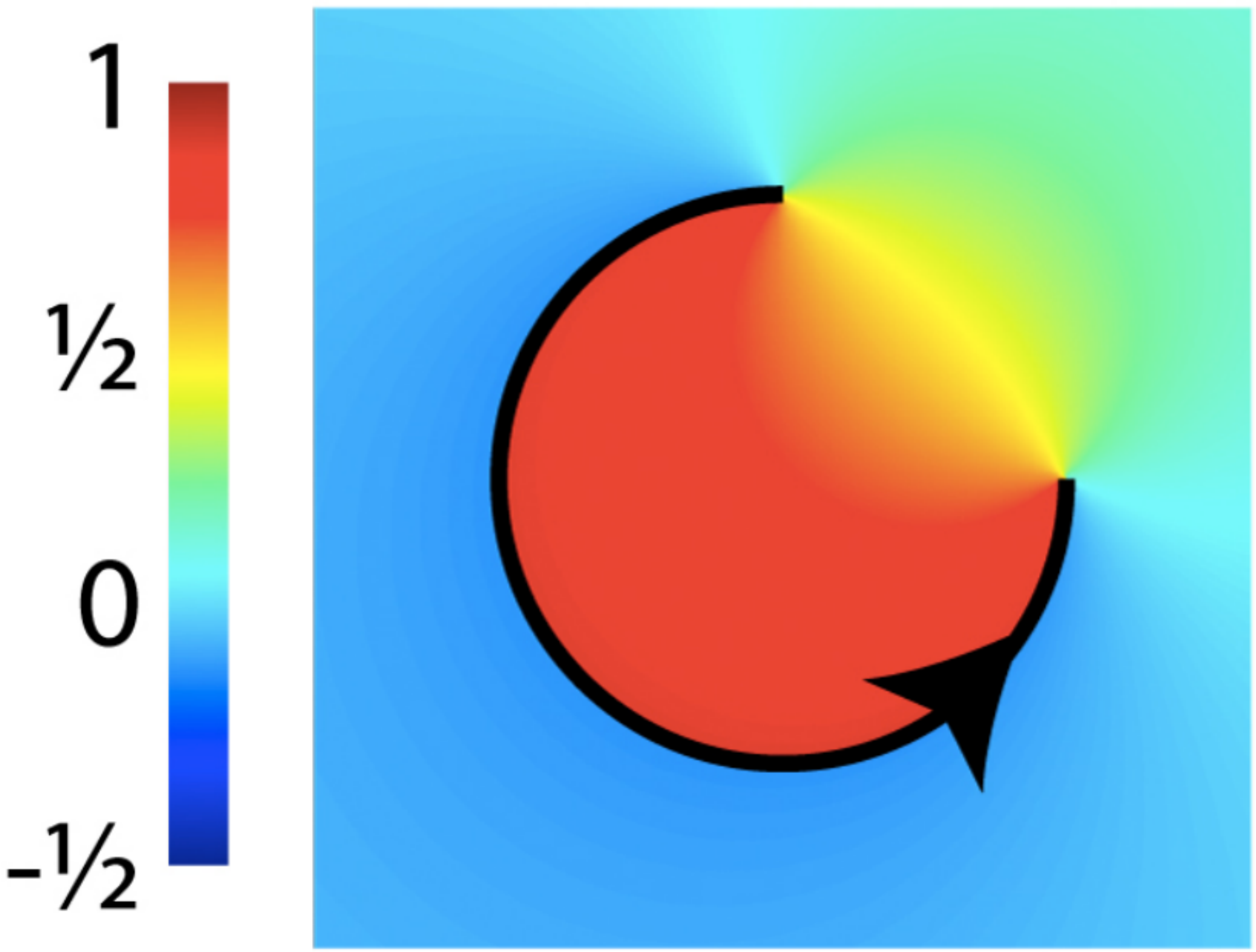
Sum of over 2π

Surface:

Sum of Solid Angles over 4π

Generalized Winding Numbers

- Outside: **0**
- Inside with certainty: **1**
- Natural inside-outside threshold: **0.5**



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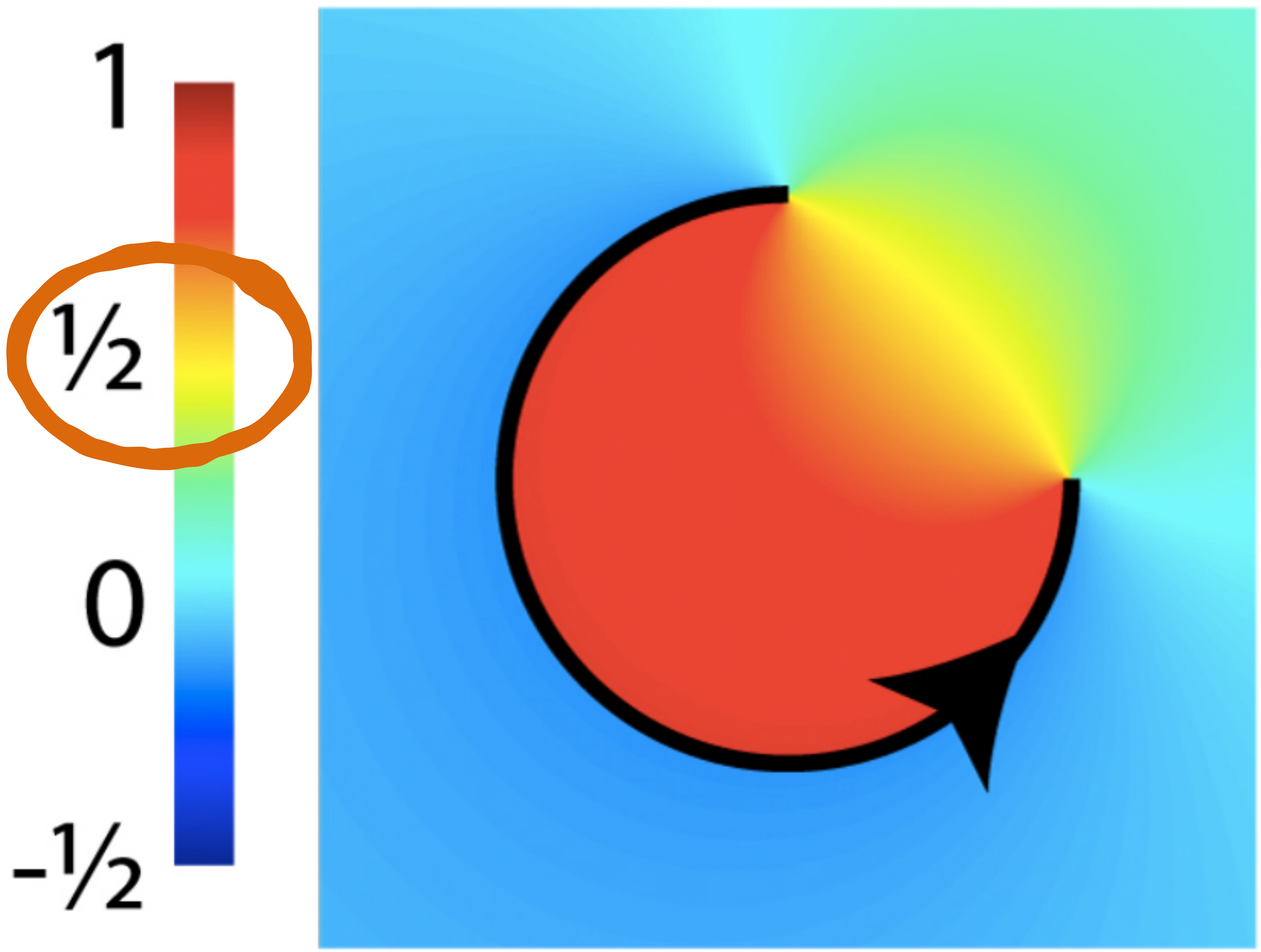

Generalized Winding Numbers Papers

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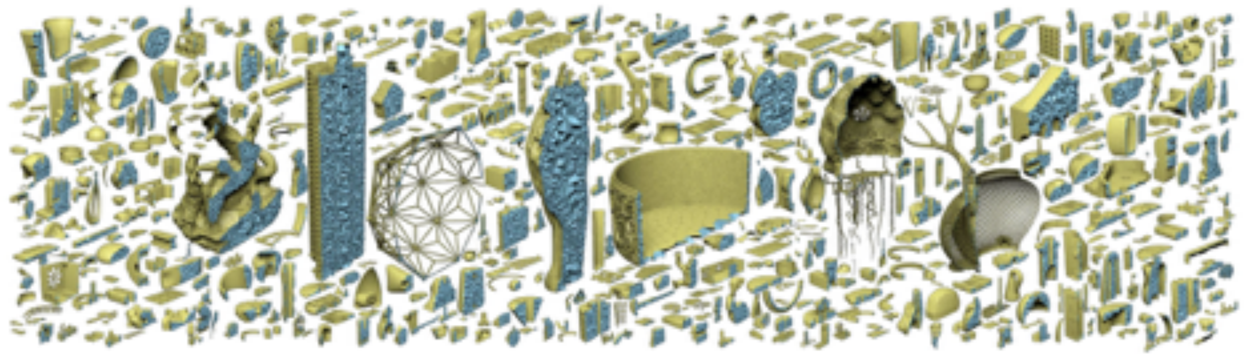
Tetrahedral Meshing in the Wild

YIXIN HU, New York University
QINGNAN ZHOU, Adobe Research
XIFENG GAO, New York University
ALEC JACOBSON, University of Toronto
DENIS ZORIN, New York University
DANIELE PANOZZO, New York University



Mesh Arrangements for Solid Geometry

Qingnan Zhou[1] Eitan Grinspun[2] Denis Zorin[1] Alec Jacobson[2]

[1]New York University [2]Columbia University

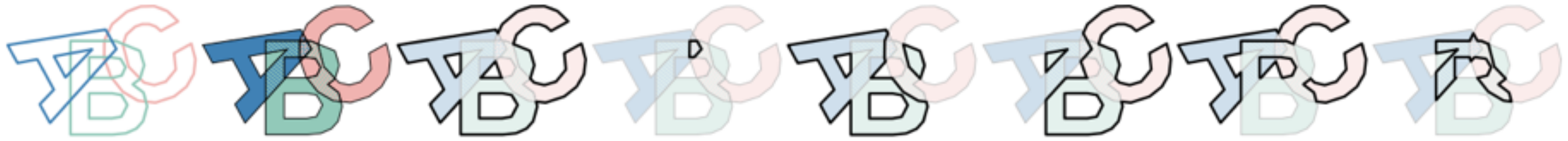


Figure 1: *Our method takes as input any number of meshes (three shown in this 2D illustration). We resolve intersections and assign a winding number vector to every delineated cell. Different boolean results are extracted according to these winding number vectors.*

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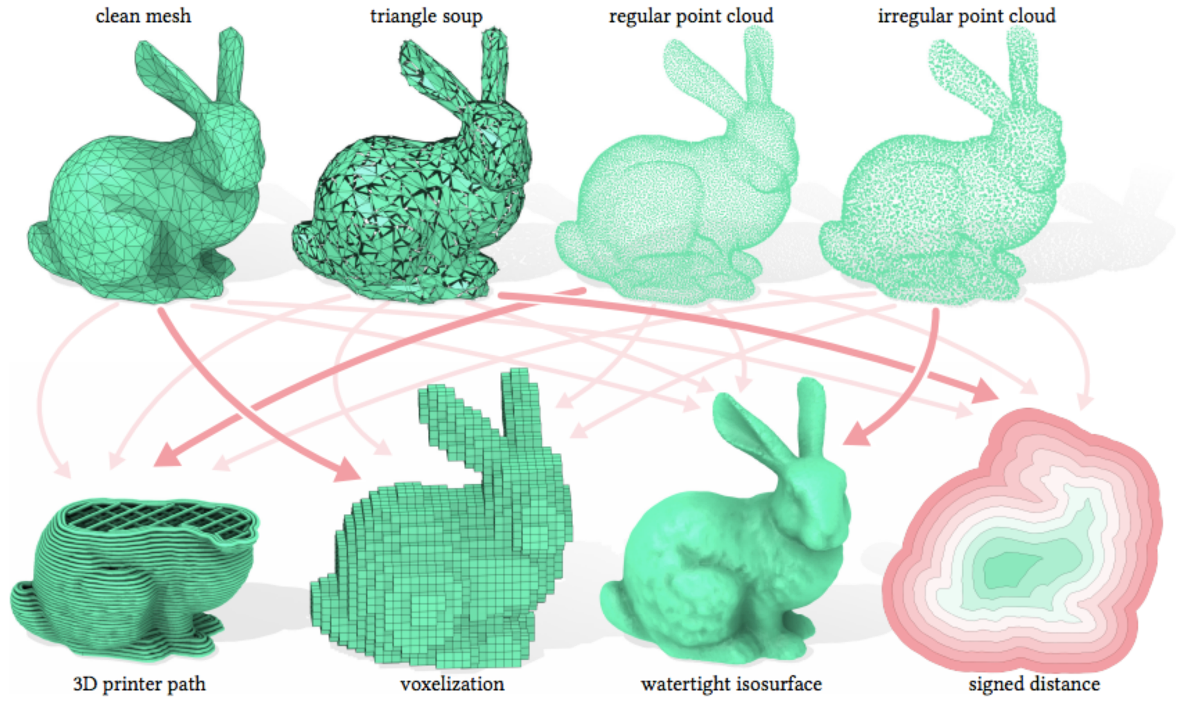

Fig. 1. In this paper, we further generalize the winding number to point clouds and propose a hierarchical algorithm for fast evaluation (up to 1000× speedup). This enables efficient answers to inside-outside queries for a wider class of shape representations (top) during a variety of tasks (bottom).

SIGGRAPH'13 **SIGGRAPH'16** **SIGGRAPH'18**

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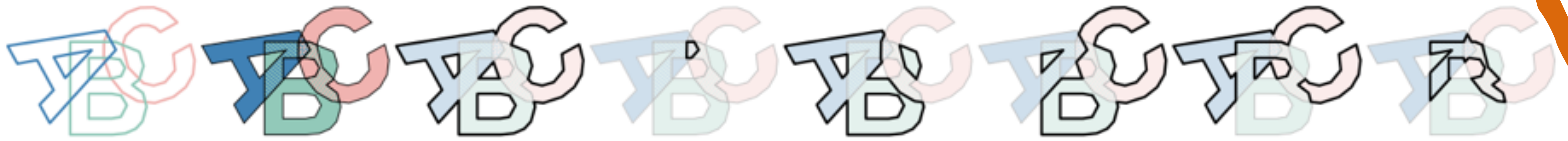

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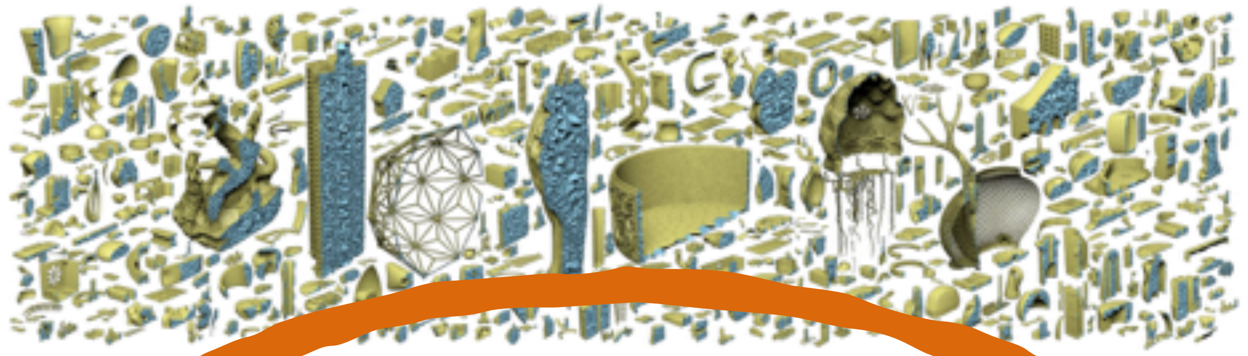

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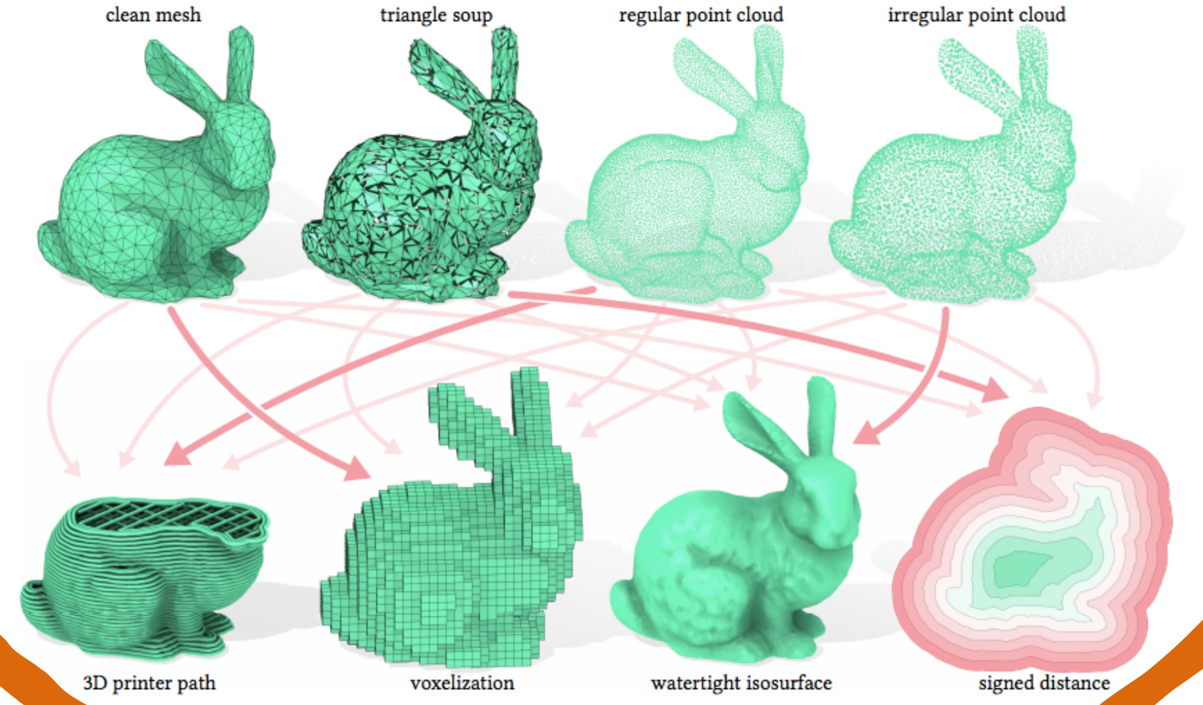

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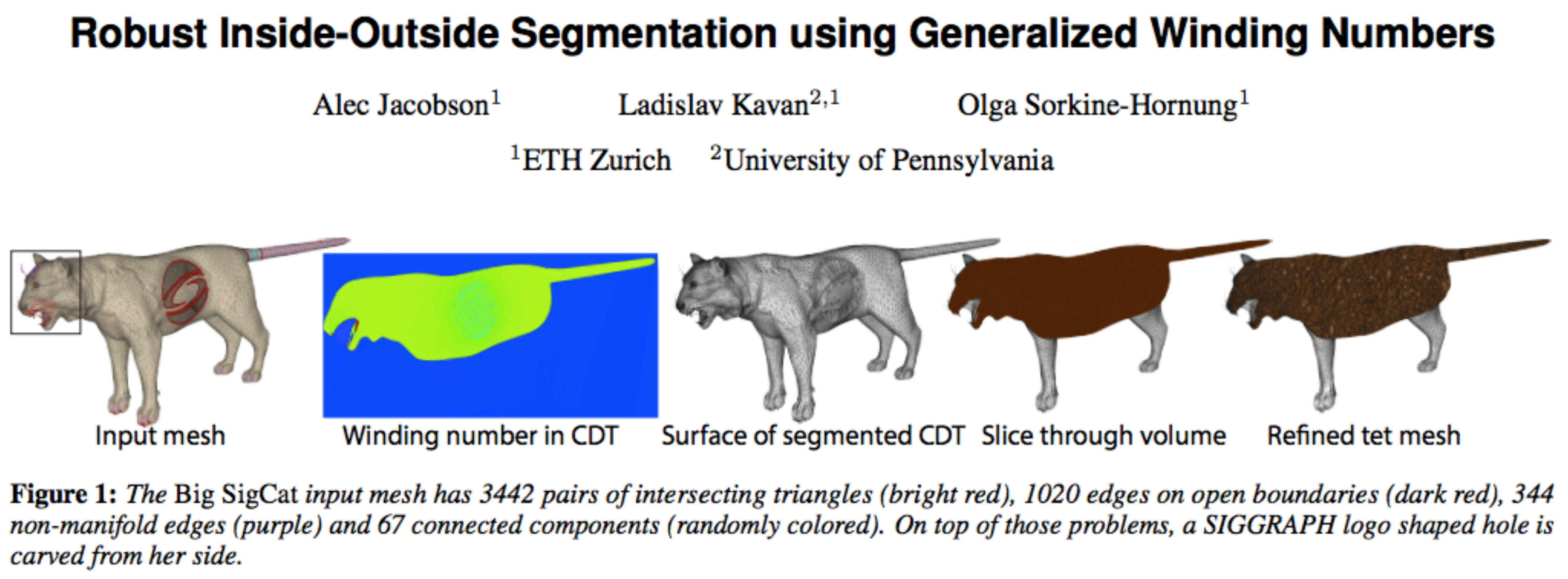

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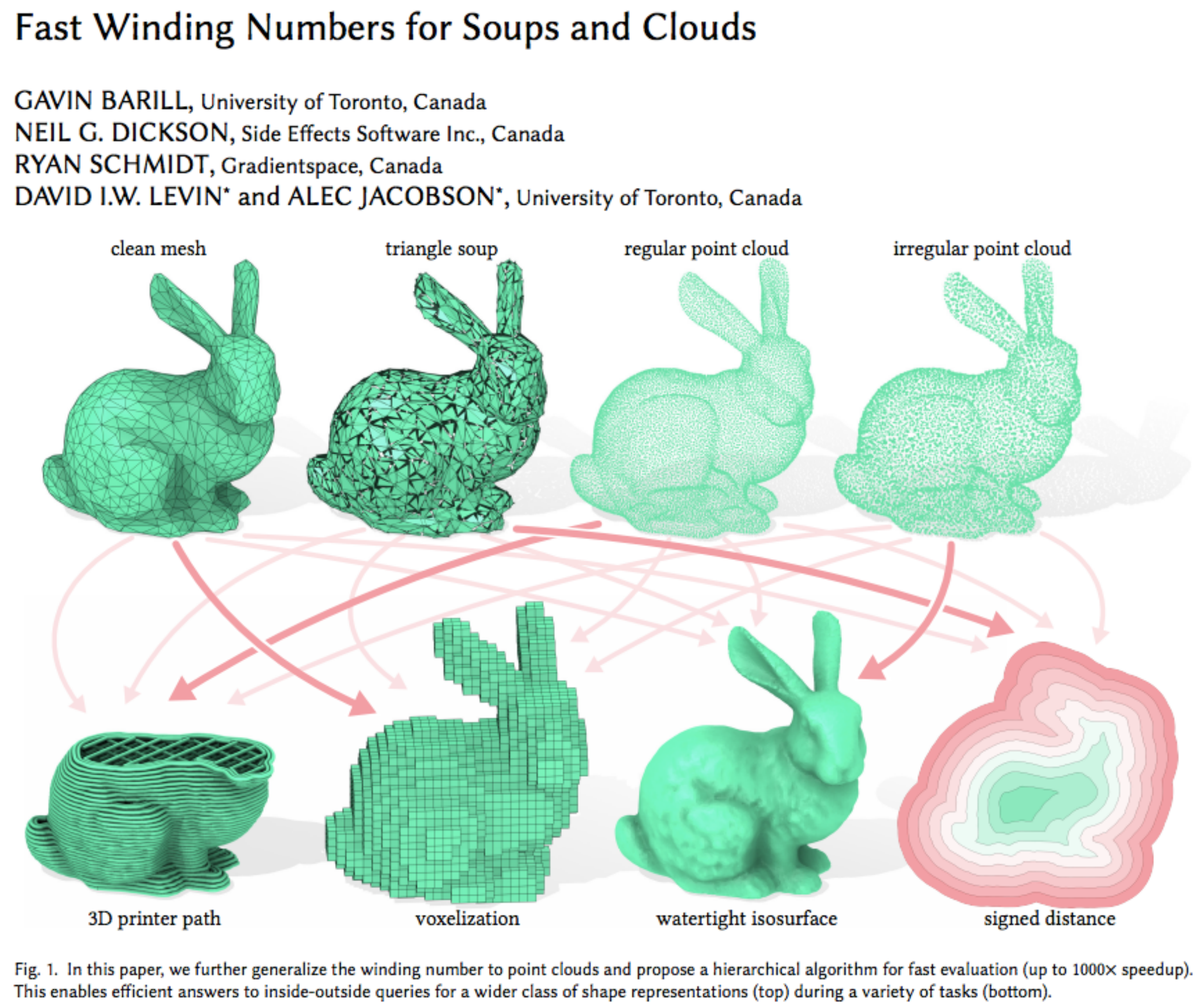
Speed up



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[Jacobson et al. 2013]

- Runs on tetrahedrons
- Divide-and-conquer based
- Could degenerate to naive sum (slow) for point cloud data

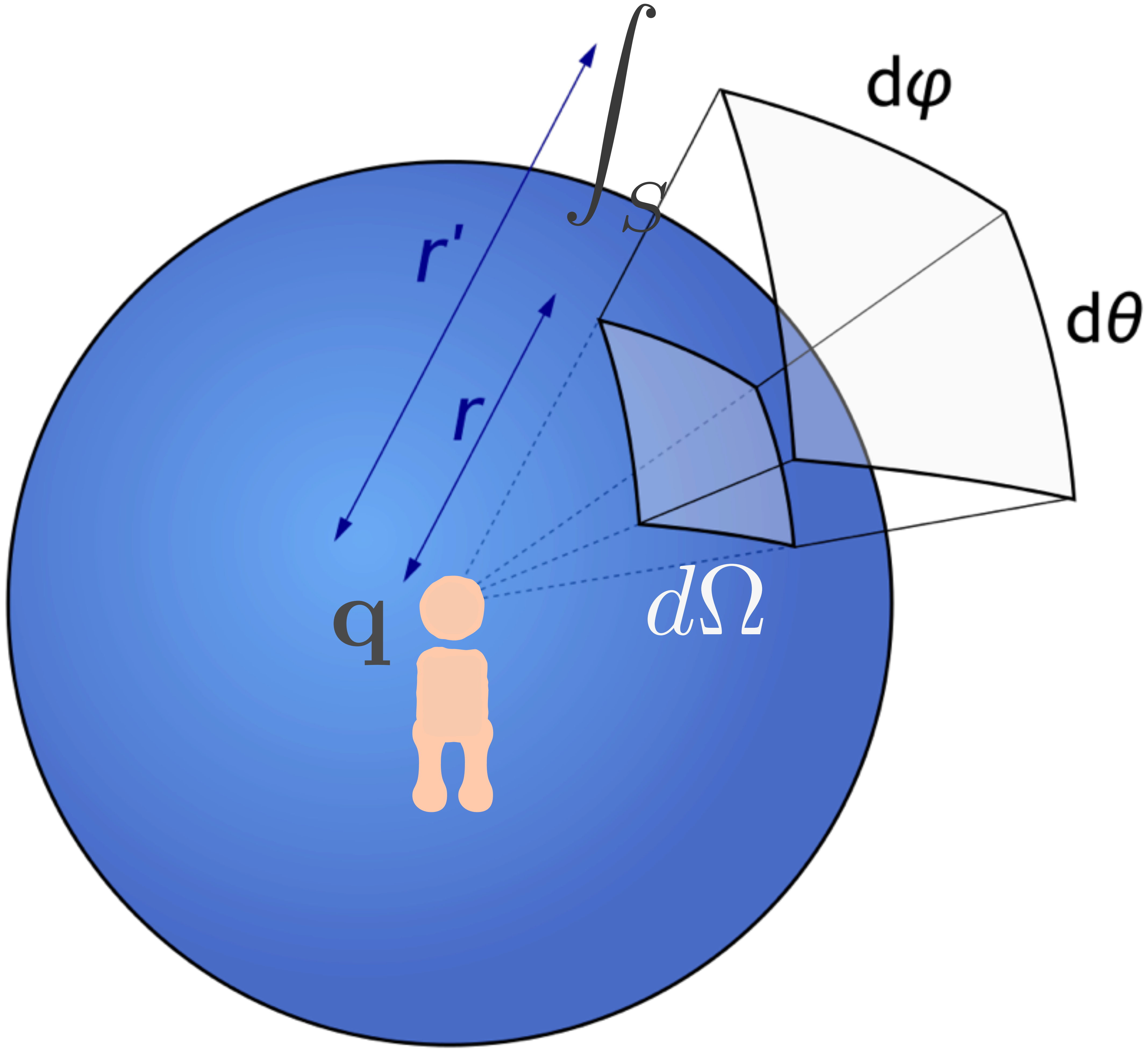
[Barill et al. 2018]

- Runs on arbitrary query points
- Tree based abstraction
- Can handle point clouds

Winding Numbers from Two Viewpoints

Looking out to the surface from a given query point:

$$w(\mathbf{q}) = \frac{1}{4\pi}\int_S d\Omega(\mathbf{q})$$

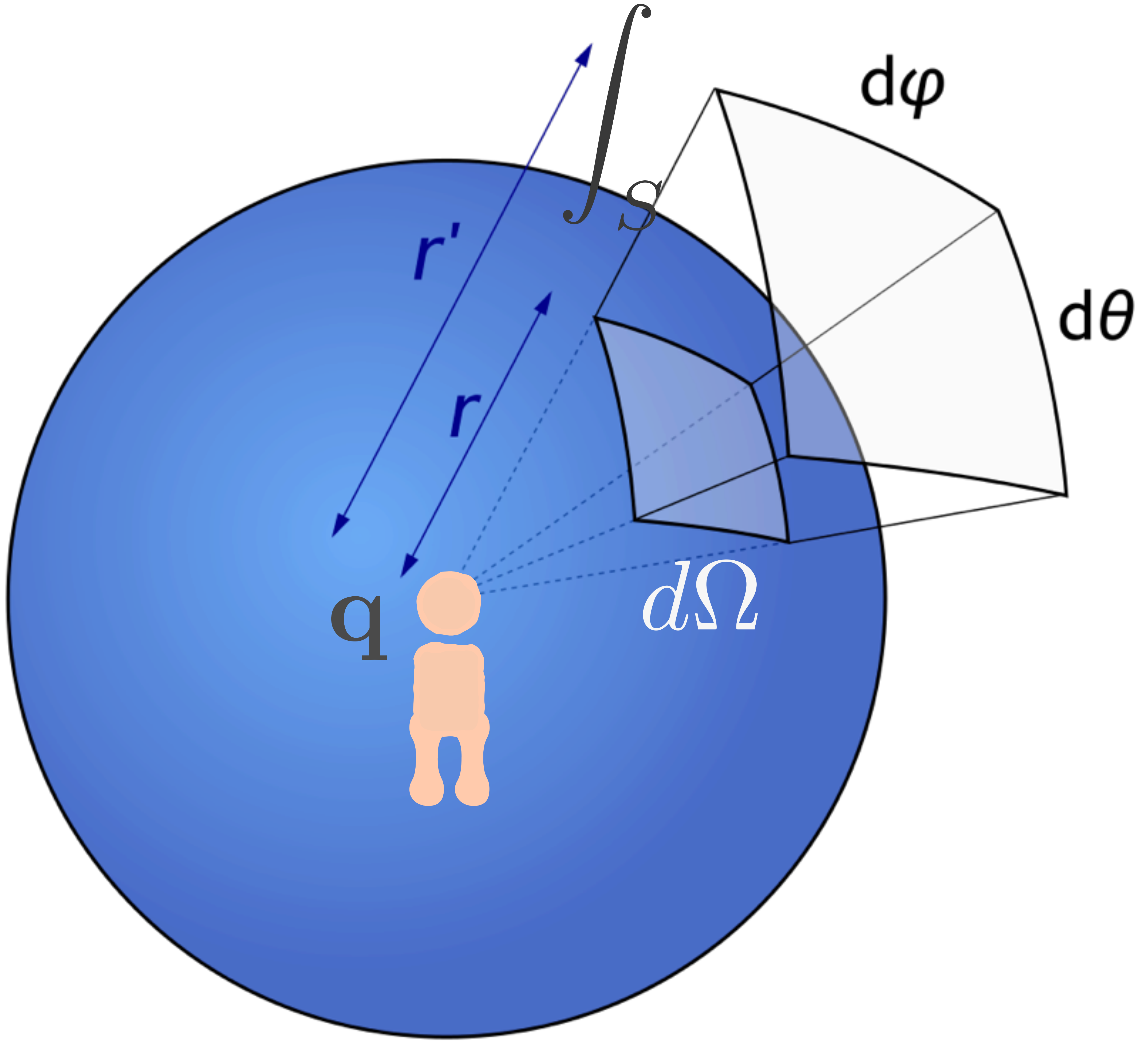


https://www.kisspng.com/png-solid-angle-sphere-cone-steradian-angle-665787/

Winding Numbers from Two Viewpoints

Expand the solid angle:

$$\frac{1}{4\pi}\int_S d\Omega(\mathbf{q})$$
$$= \frac{1}{4\pi}\int_S \cos\beta\, dA$$



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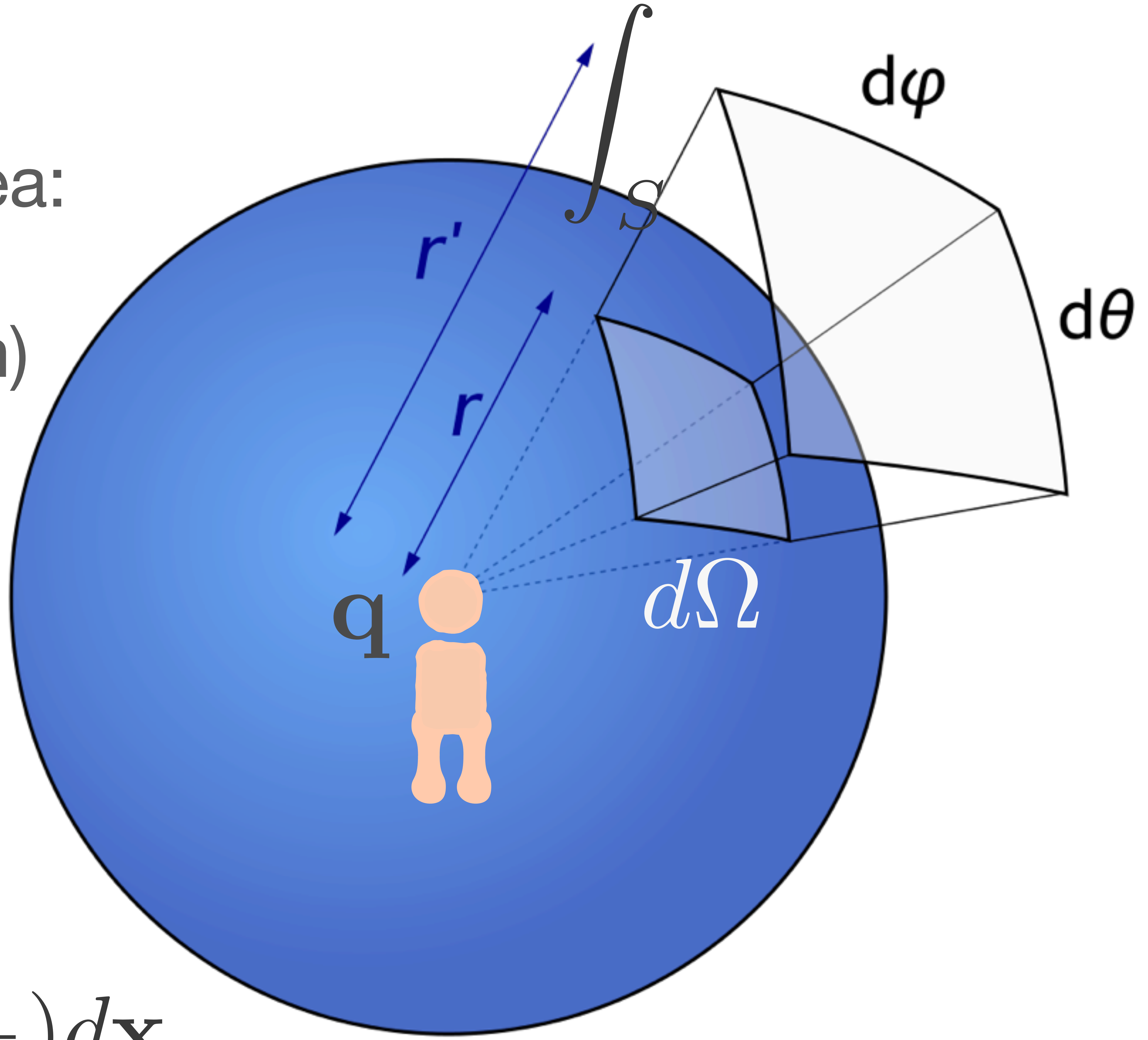
Winding Numbers from Two Viewpoints

Given an arbitrary point x on the surface, plug in the dot product and the scaled area: (This is the point cloud result, for polygon soup, replace the solid angle computation)

$$\frac{1}{4\pi}\int_S d\Omega(\mathbf{q})$$

$$= \frac{1}{4\pi}\int_S \cos\beta dA$$

$$= \frac{1}{4\pi}\int_S (\frac{\mathbf{x}-\mathbf{q}}{||\mathbf{x}-\mathbf{q}||}\cdot\mathbf{n})(\frac{1}{||\mathbf{x}-\mathbf{q}||^2})d\mathbf{x}$$



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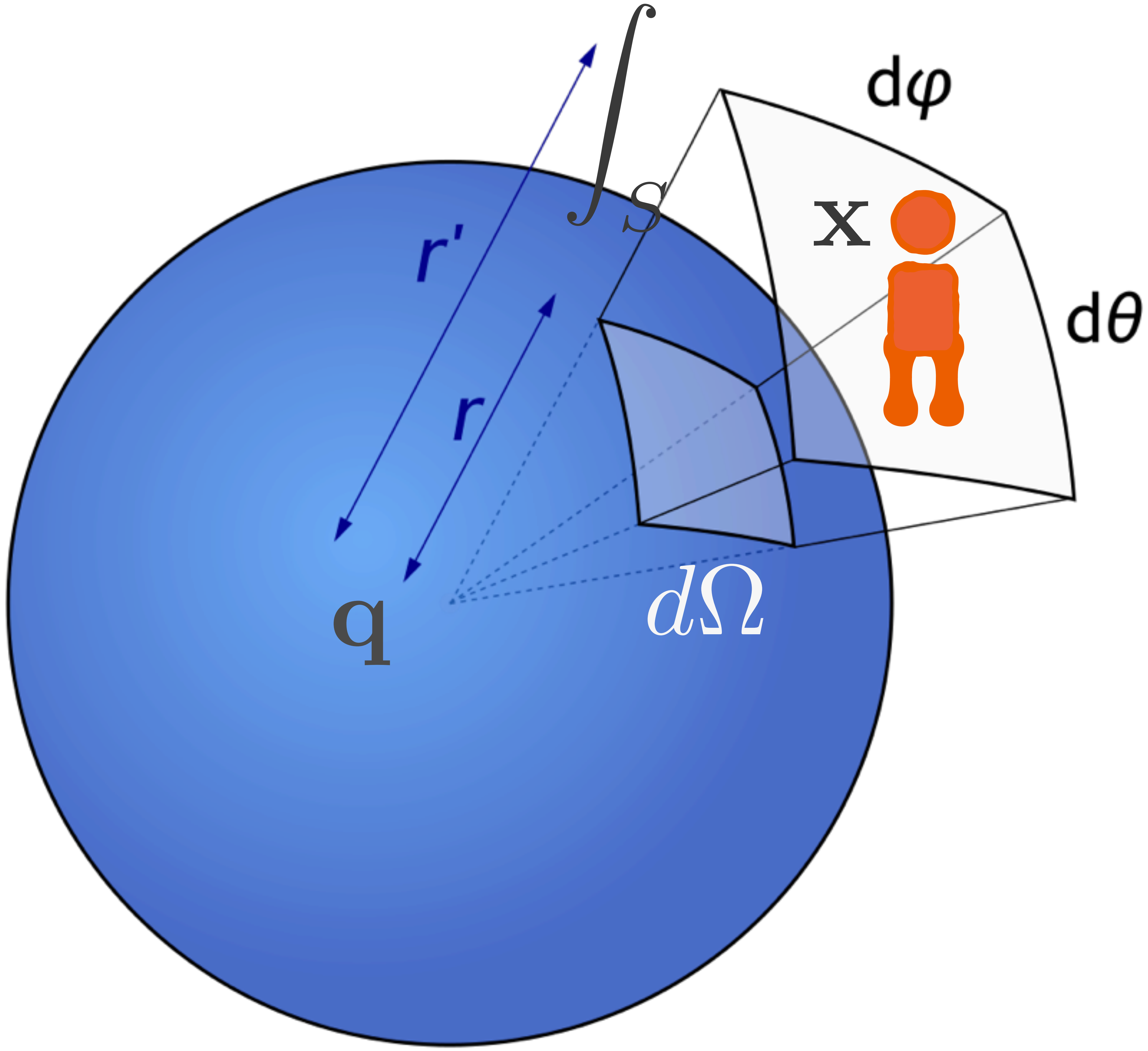
Winding Numbers from Two Viewpoints

From the view of the point on the surface, its contribution to the generalized the winding number is:

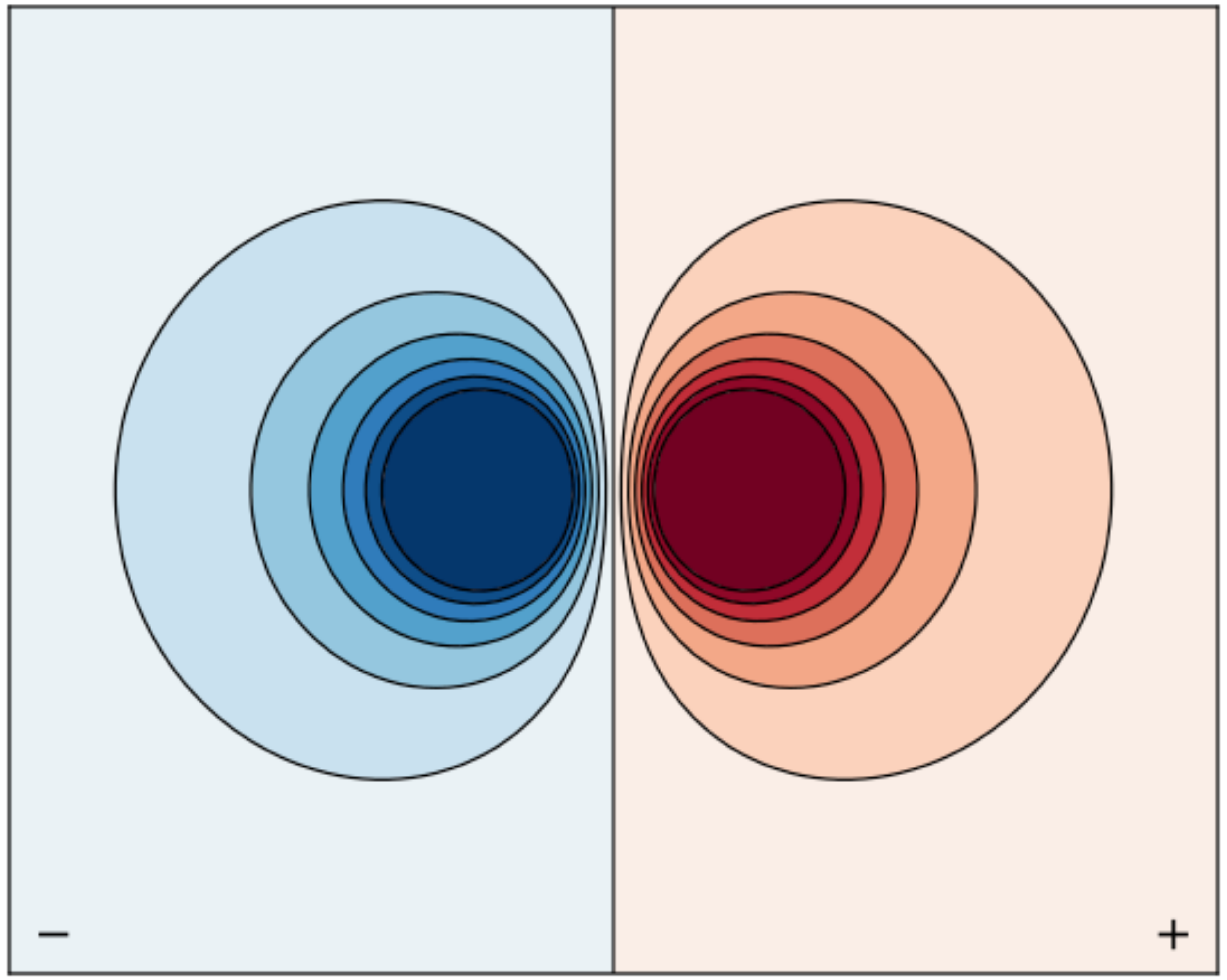
$$\frac{(\mathbf{x}-\mathbf{q})\cdot\mathbf{n}}{4\pi||\mathbf{x}-\mathbf{q}||^3}$$



https://www.kisspng.com/png-solid-angle-sphere-cone-steradian-angle-665787/

Dipoles on the Surface

They observed that this contribution function is actually a **dipole** (two oppositely charged points close together), which looks like this:



https://en.wikipedia.org/wiki/Dipole#/media/File:Dipole_Contour.svg

Large Dipole on the Surface

They also observed that looking from a far distance, a cluster of dipoles is similar to a single large dipole.

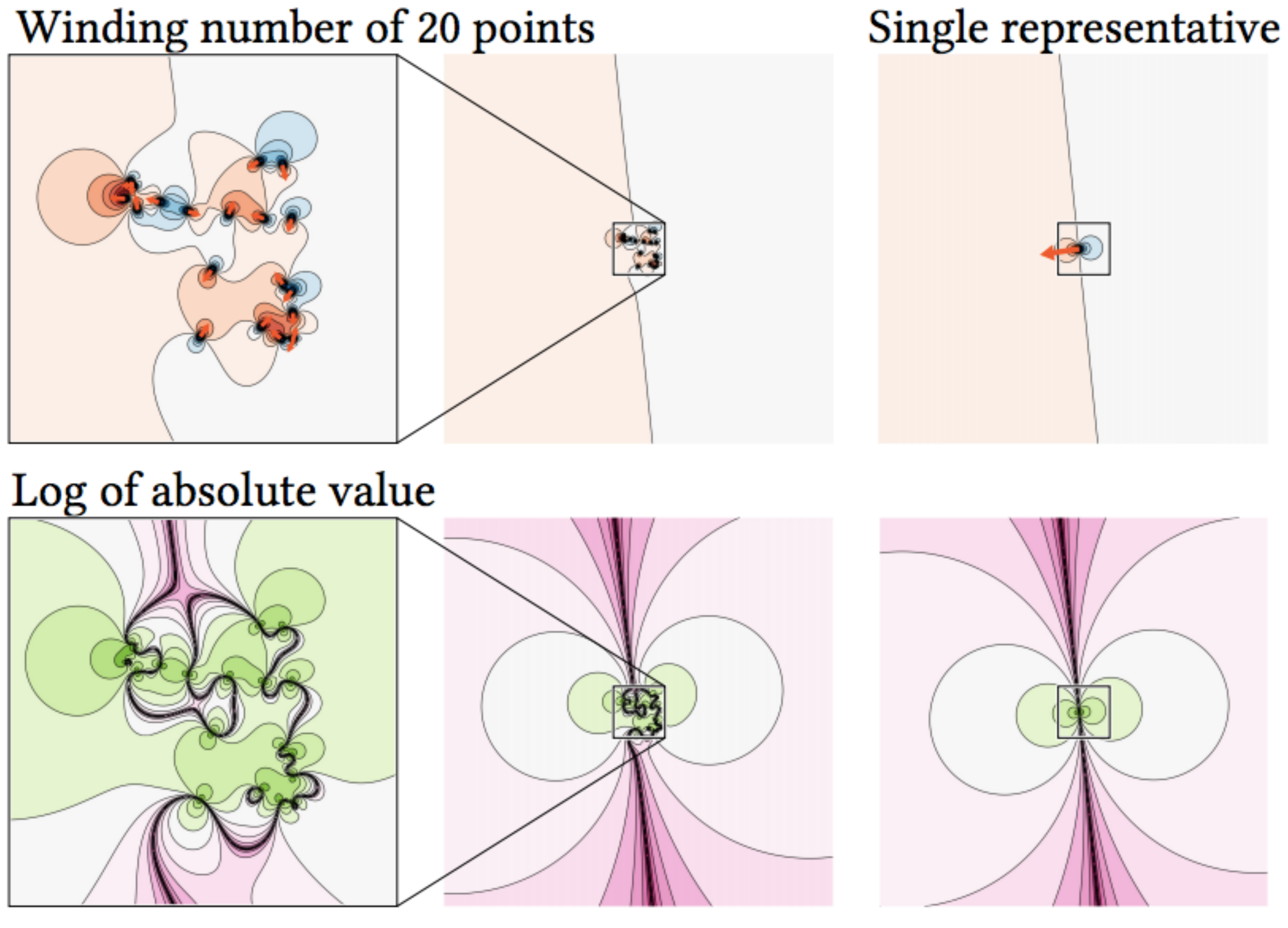


Fig. 6. A cluster of 20 dipoles has an intricate winding number field nearby (left), but far away their function is quite tame (middle) and well approximated by a single, *stronger* dipole (right).

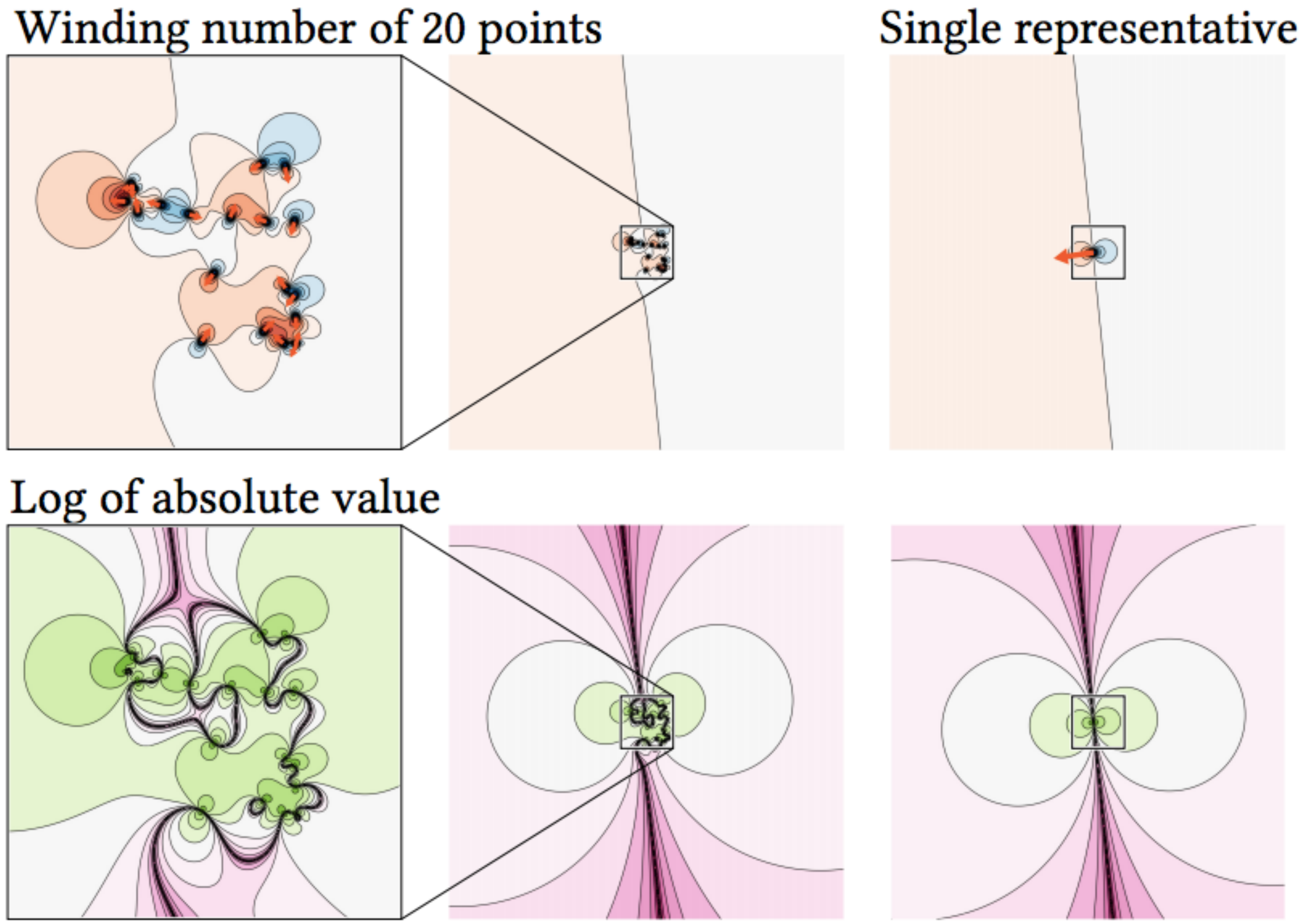
Large Dipole on the Surface

Thus, the original weighted sum can be replaced by a combined dipole:

$$w(\mathbf{q}) = \sum_{i=1}^{m} a_i \frac{(\mathbf{p}_i - \mathbf{q}) \cdot \hat{\mathbf{n}}_i}{4\pi \|\mathbf{p}_i - \mathbf{q}\|^3} \approx \frac{(\tilde{\mathbf{p}} - \mathbf{q}) \cdot \tilde{\mathbf{n}}}{4\pi \|\tilde{\mathbf{p}} - \mathbf{q}\|^3} =: \tilde{w}(\mathbf{q})$$

$$\tilde{\mathbf{n}} = \sum_{i=1}^{m} a_i \hat{\mathbf{n}}_i, \quad \tilde{\mathbf{p}} = \frac{\sum_{i=1}^{m} a_i \mathbf{p}_i}{\sum_{i=1}^{m} a_i},$$

Mean weighed normal **Mass center**



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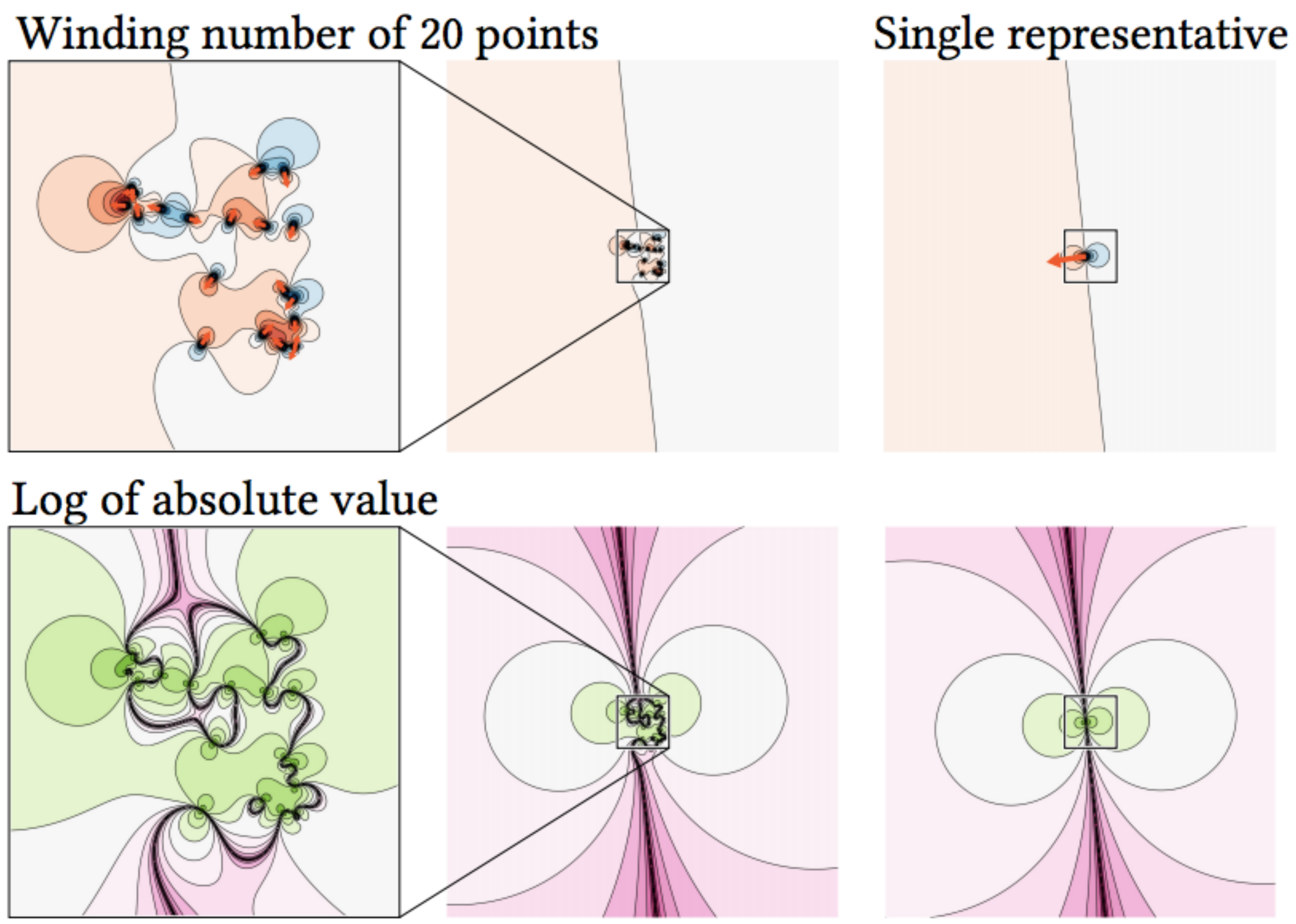
Large Dipole on the Surface

They also observed that this approximation is the first term of the Taylor expansion.

In this paper, they use the second-degree approximation.

For details, see the paper.



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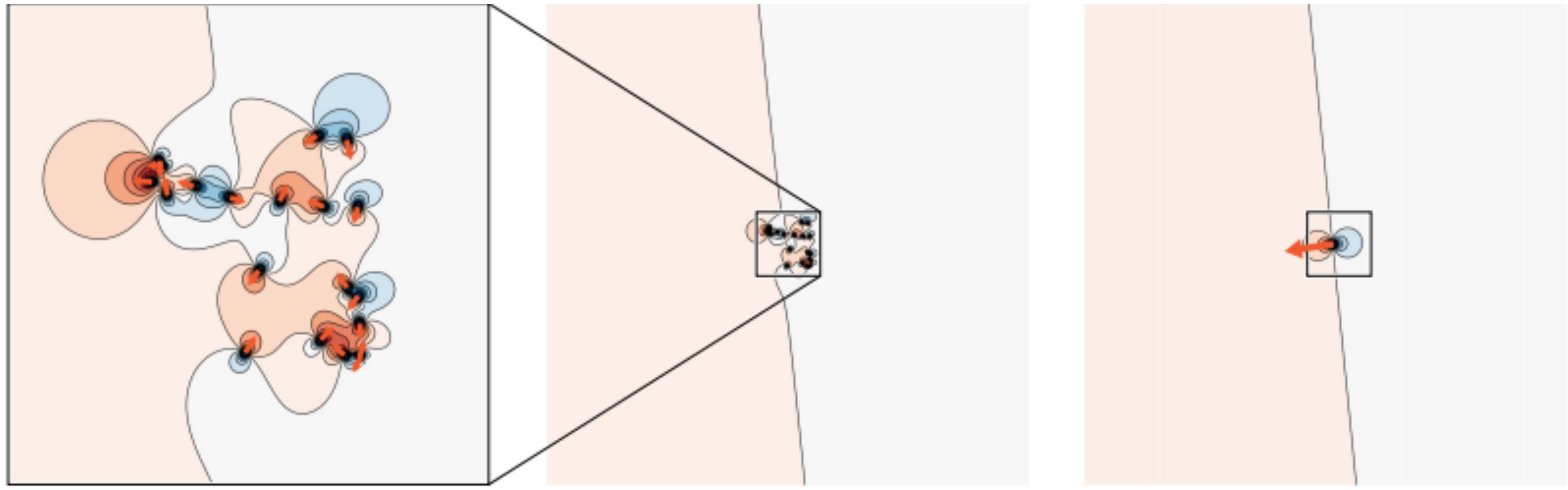
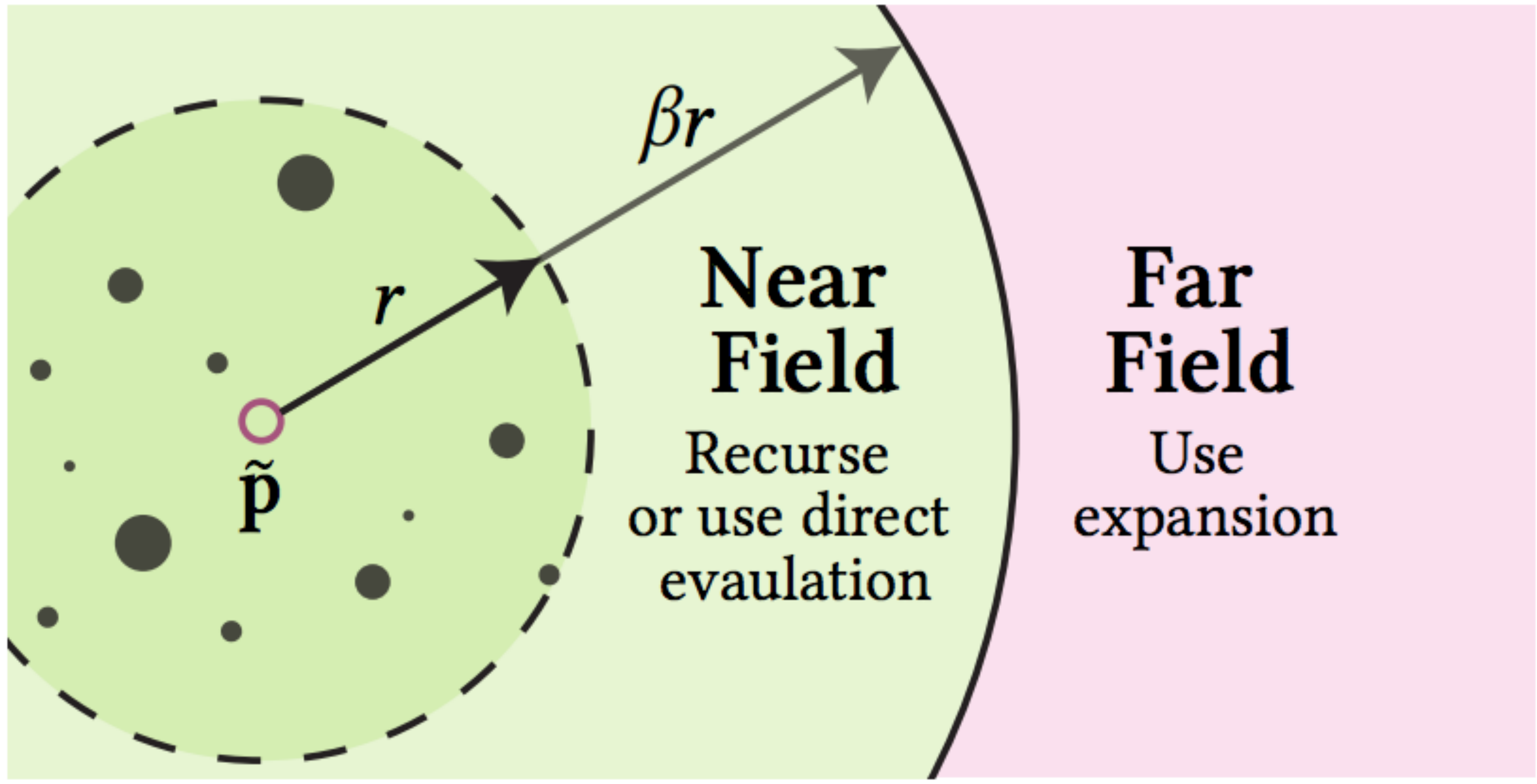
Tree Algorithm

Build a BVH with approximate dipole at each level (pre-computation)

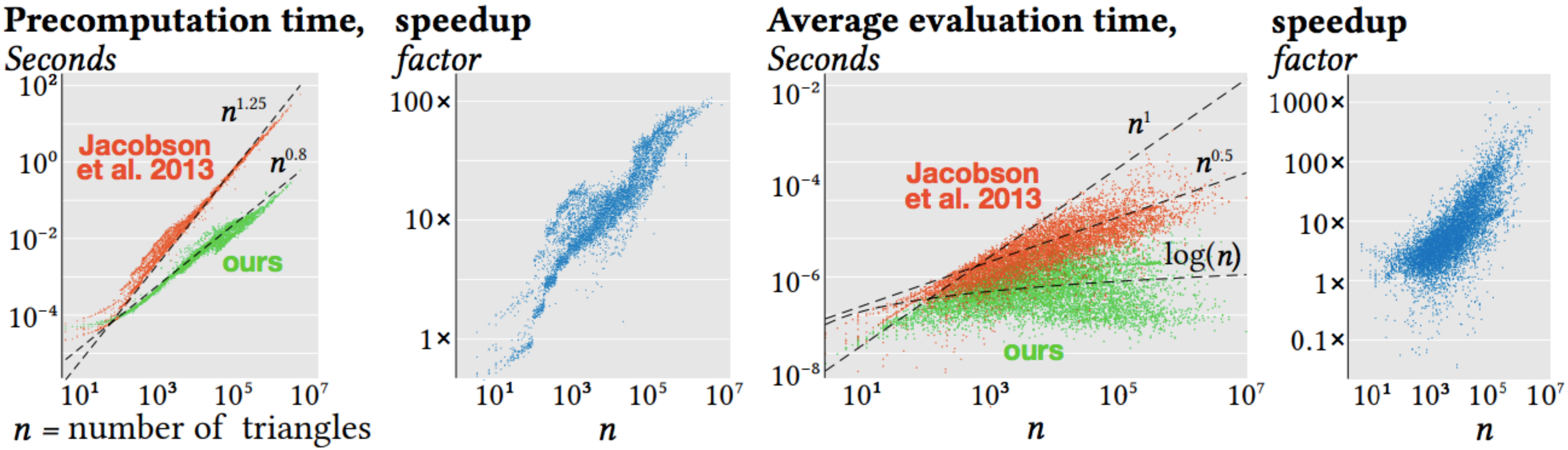


Sum the winding number by querying the tree from the root

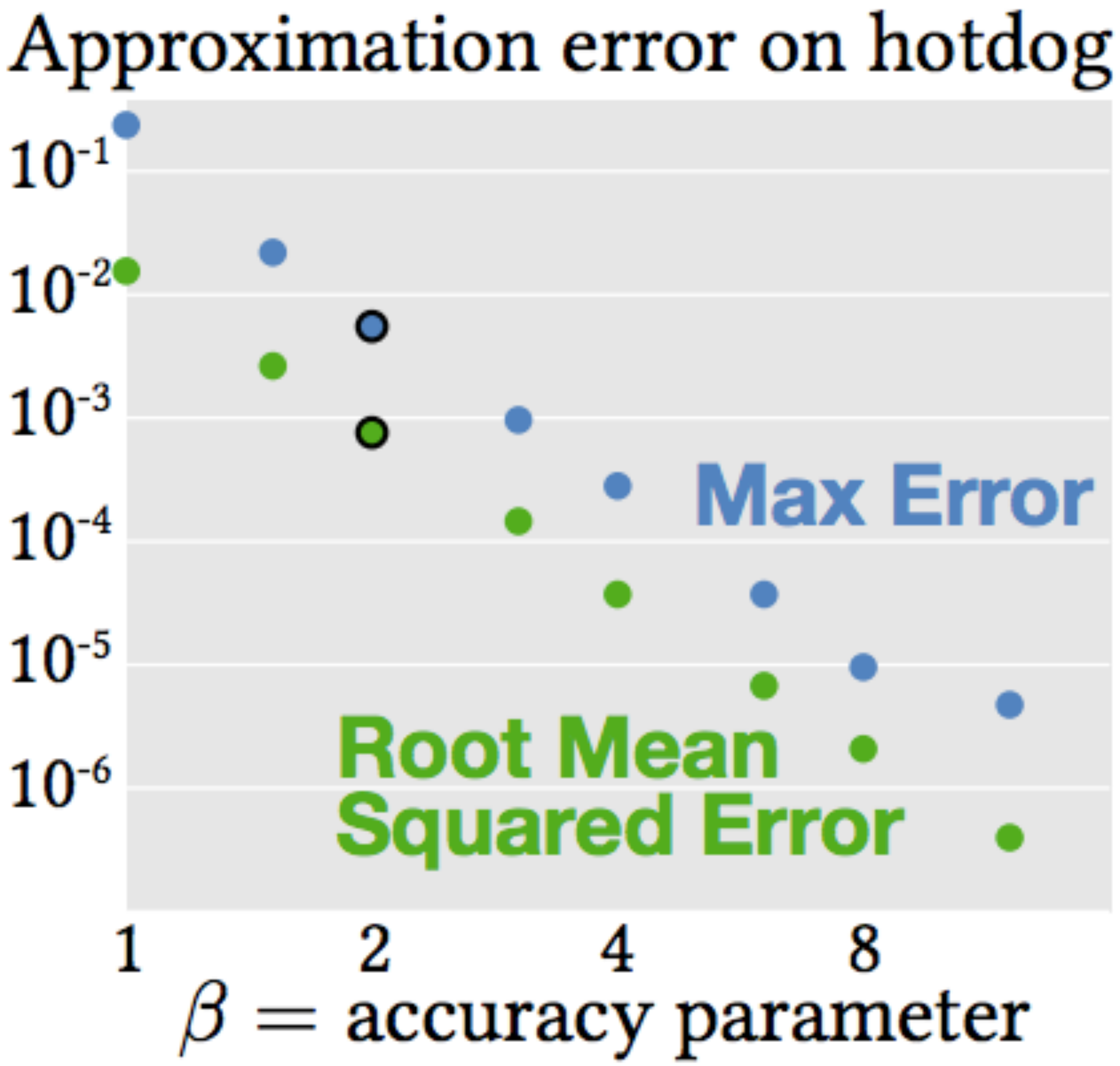
$$O(nm) \rightarrow O(n \log m)$$



Results: Runtime

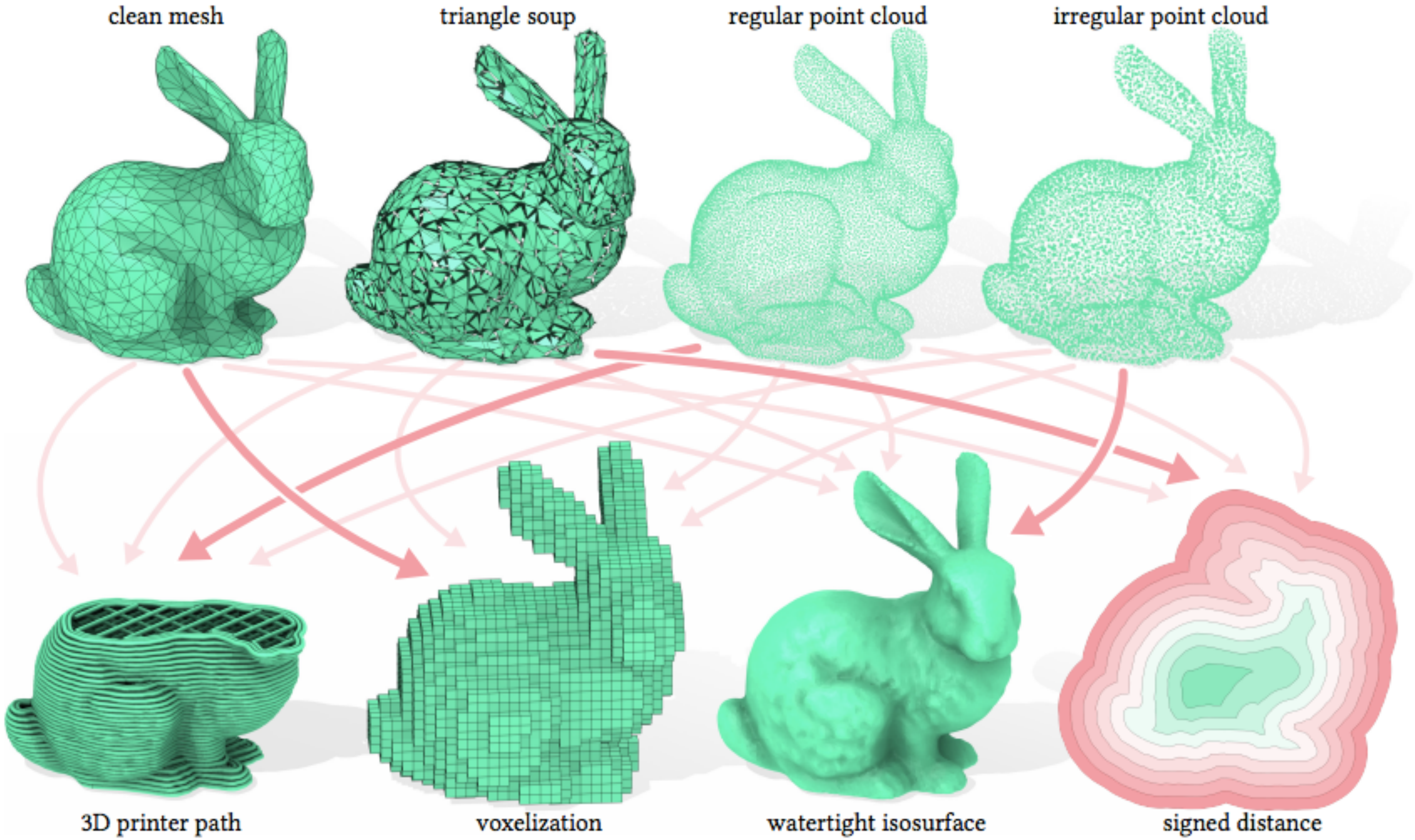


Results: Accuracy



Results: Applications

Generalized winding number can be seen as an implicit representation of the mesh and it’s easy to transfer to other representations (isosurface, voxelization, cross-section etc).



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Limitations

1. Current generalized winding number doesn't consider noisy data (all test point clouds in the paper are quite clean).

2. The input data needs to have correct orientations.

(3. The to-be-reconstructed surface needs to be mostly covered by the input data. So it doesn't apply to curve network.)

Comments

1. Good paper with good SIGGRAPH presentation. It contains approximation methods from two different “fields”:

    - Continuous-mathematic approximation (Taylor expansion)

    - Algorithmic approximation (BVH tree)

2. Possible applications beyond those in paper?

    - I’m interested to see how it works on figuring out correct topology of 2D line drawings. (It may have issues with the noise.)

THANKS!
DISCUSSIONS & COMMENTS

Winding!

http://irunlikeme.blogspot.com/2011/03/want-to-improve-your-running-run-smart.html